Development of a High-Order Strand Solver for Helios

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Presented by:
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Outline

• Background & Motivation
• High-order Solver formulation
• Preliminary Results
• Summary and Conclusions
Background

• Complexities in high fidelity rotary-wing aeromechanics prediction
  – Complex geometries
  – High-Re wall-bounded viscous flow
  – Wake resolution
  – Strong aero-structure coupling, particularly blade twist from pitching moment
Issue #1: Automation

- Rotorcraft CFD steps

How do we enable skilled rotorcraft engineers to use high-fidelity CFD tools without forcing them to become grid generation experts?
Issue #2: Accuracy

- Lower order near-body solver limits ability to resolve tip rollup

Comparing PIV-measured vorticity to computation

HART-II

Tip vortex dissipation

Lim et al AHS Forum'2012
Issue #3: Speed

- Near-body solver is the most expensive portion of the simulation

Model V22 Hover - 128 procs

Two-thirds total cost

UH60 Fwd Flight – 512 procs
Strand Technology Addresses these Issues

• Automation
  – Near-body strands grown directly from surface tessellation
  – Cartesian off-body resolution adjusted according by available compute resources
  – *Strand-Cartesian volume mesh generated automatically at runtime*

• Accuracy & Efficiency
  – High-order solver formulation that takes advantage of strand data structure
  – Fast and scalable domain connectivity
  – Structured data ensures fast numerics
  – *4<sup>th</sup> order solutions at only 1.5X cost of 2<sup>nd</sup> order solutions*
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• **Strand normal direction:**
  **High Order Finite Differences**
  – Summation by parts with variable coefficients
  – Reduces to finite difference at interior
  – Satisfies stability and accuracy constraints

• **Unstructured streamwise direction:**
  **High Order Flux corrections**
  – Achieves high order through truncation error cancellation of finite volume scheme
  – Layers coupled via source term containing derivatives in strand direction
Solver

RANS-SA equations

\[ \frac{\partial Q}{\partial t} + \frac{\partial F_j}{\partial x_j} - \frac{\partial F_j^v}{\partial x_j} = S \]

\[
Q = \begin{pmatrix}
\rho \\
\rho u_i \\
\rho e \\
\rho \tilde{v}
\end{pmatrix}, \quad F_j = \begin{pmatrix}
\rho u_j \\
\rho u_i u_j + p \delta_{ij} \\
\rho h u_j \\
\rho \tilde{v} u_j
\end{pmatrix}, \quad F_j^v = \begin{pmatrix}
0 \\
\sigma_{ij} \\
\frac{n}{\sigma} \frac{\partial \tilde{v}}{\partial x_j} \\
q_j
\end{pmatrix}, \quad S = \begin{pmatrix}
0 \\
0 \\
0 \\
(P - D + C_{b2} \rho \frac{\partial \tilde{v}}{\partial x_k} \frac{\partial \tilde{v}}{\partial x_k})
\end{pmatrix}
\]

\[ P = \begin{cases}
\rho C_{b1} (1 - f_{i2}) \tilde{S} \tilde{v}, & \tilde{v} \geq 0 \\
\rho C_{b1} (1 - C_{t3}) \Omega \tilde{v} & \tilde{v} < 0
\end{cases} \quad D = \begin{cases}
\rho \left( C_{\omega 1} f_{\omega} - \frac{C_{b1}}{\kappa^2} f_{i2} \right) (\frac{\tilde{v}}{\tilde{a}})^2, & \tilde{v} \geq 0 \\
-\rho C_{\omega 1} (\frac{\tilde{v}}{\tilde{a}})^2 & \tilde{v} < 0
\end{cases} \]

- Spalart Allmaras turbulence model treatment
  - Allows negative turbulence working variable (Allmaras 2012)
  - Fully-coupled high-order treatment
Strand Mapping

- Map from physical to computational space
  - Equally-spaced sub-triangles in $r$-$s$ (streamwise) plane in computational space
    - Cubic or quadratic sub-triangles
  - Stretched node distribution in $\eta$ (normal) direction mapped to equal-spaced distribution in computational space
  - Surface triangles treated as cubic or quadrilateral elements
Flux Correction Scheme

- **Finite Volume flux balance**

\[
F^h_{0i} = \frac{1}{2} (F^L_{0i} + F^R_{0i}) - \frac{1}{2} \left| A(Q^R, Q^L) \right| (Q^R - Q^L)
\]

- **Compute left/right fluxes such that truncation error of each cancels when added together**

\[
F^L_{0i} = F^h_0 + \frac{1}{2} \Delta r^T_{0i} \nabla^h F^h_0
\]

\[
F^R_{0i} = F^h_i - \frac{1}{2} \Delta r^T_{0i} \nabla^h F^h_i
\]

- **Advantages:**
  - Able to leverage finite volume techniques (shock capturing, efficient solvers, etc.)
  - no high-order quadrature or least squares reconstruction
  - builds on existing infrastructure
Extend flux corrected schemes to turbulent flows on high aspect ratio strand grids.

**Flux Correction Schemes**

**Previous Work**

*Katz and Sankaran*  

- Subsonic  
  **NACA 0012**  
  (steady, inviscid)
  - Finite Volume
  - Flux Corr

- Fin Vol - 8619 nodes  
- Flux Corr - 4620 nodes

*Pincock and Katz,*  
**AIAA-2013-2566**

- Shedding square  
  (M=0.1, Re=250, ~3000 nodes)
  - Finite Volume
  - Flux Corr
Strand Direction Coupling

- Treat strand direction derivatives as source term to preserves flux correction accuracy

\[
\frac{\partial Q}{\partial \tau} + \frac{\partial F}{\partial r} + \frac{\partial G}{\partial s} - \frac{\partial F_{i\nu}}{\partial r} - \frac{\partial G_{i\nu}}{\partial s} = \tilde{S},
\]

\[
\tilde{S} \equiv \hat{S} - \frac{\partial \hat{Q}}{\partial t} - \frac{\partial \hat{H}}{\partial \eta} + \frac{\partial \hat{H}_{i\nu}}{\partial \eta}.
\]

\[
\frac{\partial F}{\partial x} = \tilde{S}, \quad \tilde{S} \equiv S - D_y G
\]

\[
D_y G = \frac{\partial G}{\partial y} + O(h^p)
\]

Source Treatment

\[
\frac{1}{\Delta x_{i,j}} \left[ \left( F_{i+\frac{1}{2},j} - F_{i-\frac{1}{2},j} \right) - S_{i,j}^h \right] = \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} - S
\]

cancels

\[
\frac{\Delta x_{i+\frac{1}{2}} + \Delta x_{i-\frac{1}{2}}}{24 \Delta x_i} \left( F_{3x} - \tilde{S}_{2x} \right) + O(h^p) + O(h^q) + O(h^3).
\]

No Source Treatment

\[
\frac{1}{\Delta x_{i,j}} \left[ \left( F_{i+\frac{1}{2},j} - F_{i-\frac{1}{2},j} \right) + D_y G_{i,j} - S_{i,j}^h \right] = \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} - S
\]

does not cancel

\[
- \frac{\Delta x_{i+\frac{1}{2}} + \Delta x_{i-\frac{1}{2}}}{24 \Delta x_i} \left( F_{3x} - S_{2x} \right) + O(h^p) + O(h^q) + O(h^3).
\]
Strand Direction

- High order achieved in strand direction through finite differences
  - Summation by parts operators
  - Energy stable
  - Fernandez & Zingg, 2012; Mattsson, 2012

- Accuracy
  - 2p interior
  - p boundary
  - p+1 overall
  - Implemented p=1,2,3

3, 5, 7 point stencil
Accuracy Verification (cont)

- Inviscid terms only
- Viscous terms only
- Inviscid + Viscous

Method of Manufactured Solns

Re=100 Cylinder

3rd-4th Order achieved in tests
• **Semi-implicit Multi-grid scheme**
  – Standard FAS multigrid (Brandt, 1977)
  – LU-SGS (Yoon, Jameson) on strand layers
  – Local RK with implicit smoothing on each unstruct plane (Jameson, Mavriplis)
  – Use of triangles enables 3-element coarsening without agglomoration
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Laminar Flow Validation

- **Flow over circular cylinder**
  - Re=100

<table>
<thead>
<tr>
<th>Mesh</th>
<th>2\textsuperscript{nd} Order</th>
<th>4\textsuperscript{th} Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse (96x32)</td>
<td>steady</td>
<td>0.141</td>
</tr>
<tr>
<td>Medium (192x64)</td>
<td>0.159</td>
<td>0.165</td>
</tr>
<tr>
<td>Fine (384x128)</td>
<td>0.177</td>
<td>0.167</td>
</tr>
</tbody>
</table>

*Experiment: $St = 0.16-0.17$*
Turbulent Flow Validation

- Flow over channel bump
  - $M = 0.2$
  - $Re = 3$ Million
  - 4th order

Good correlation with NASA’s FUN3D, CFL3D

- FUN3D and CFL3D results from 1409x641 grid
- Strand grid 40X coarser
• Added moving grid terms

Moving sphere

M = 0.5

<table>
<thead>
<tr>
<th></th>
<th>CD</th>
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<tbody>
<tr>
<td>Static</td>
<td>0.2861</td>
</tr>
<tr>
<td>*Moving</td>
<td>0.3002</td>
</tr>
</tbody>
</table>

*Moving grid convergence limited by fine off-body grid extents
Dual Mesh Validation
Timing comparison w Helios

- **Same mesh, Helios vs strand solver**
  - NSU3D run on strand mesh
  - strand solver cell-centered, more DOF

<table>
<thead>
<tr>
<th></th>
<th>Helios</th>
<th>Strand</th>
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<tbody>
<tr>
<td>*Near-body</td>
<td>0.086s</td>
<td>0.604s</td>
</tr>
<tr>
<td>Off-body</td>
<td>1.82s</td>
<td>1.75s</td>
</tr>
<tr>
<td>Domain connectivity</td>
<td>6.26e-3</td>
<td>7.42e-3</td>
</tr>
</tbody>
</table>

*Dstrand solver uses more DOF than Helios*
Dual Mesh Validation
Turbulent bluff body

- **Helios implementation**
  - Strand near-body
  - SAMARC off-body

- **Bluff body separated flow over sphere**
  - $M = 0.3$, $Re = 12.0E6$
  - Dual mesh
  - Adaptive
3D Wing

- **NACA 0015 Wing**
  - Aspect Ratio = 6.6
  - $M = 0.1235$, $Re = 1.5E6$
  - Dual mesh
  - Adaptive

$V_\infty$

$M_\infty = 0.1235$, $\alpha = 12^\circ$
• **4th order** strand-based FV scheme order of magnitude cheaper than Discontinuous Galerkin (DG) methods
  – Standard finite differences in normal (strand) direction
  – Standard finite volume flux correction in streamwise directions

**Cost of 4th order scheme comparable to standard 2nd order**

*Courtesy D. Work, Utah St.*
Summary & Conclusions

- Strand technology will improve automation, accuracy, and efficiency in Helios
- In past OGS meetings we have reported on strand-specific meshing infrastructure (PICASSO) and domain connectivity (OSCAR)
- Present development focus is an efficient high-order near-body strand solver
  - Achieve up to 4<sup>th</sup>-order through a combination of finite difference and flux correction operations
  - Cost comparable to standard 2<sup>nd</sup>-order FV methods; order of magnitude cheaper than high order finite element (DG) methods
  - Accuracy on par with established FUN3D, CFL3D codes
- Anticipate initial capability release in Helios v6 (Summer 2015)
  - Multiple bodies
  - Complex geometries
Acknowledgements

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