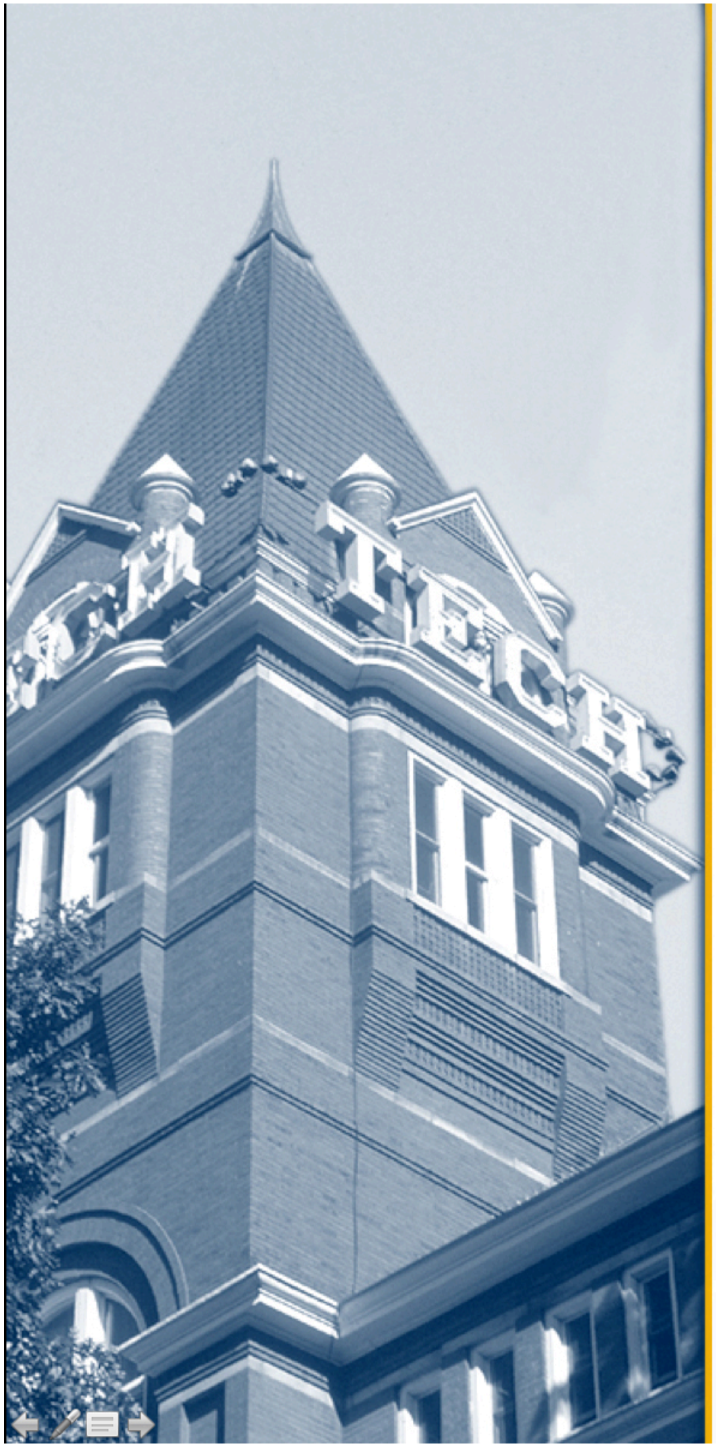



# Advanced Data Transfer Strategies for Overset Computational Methods

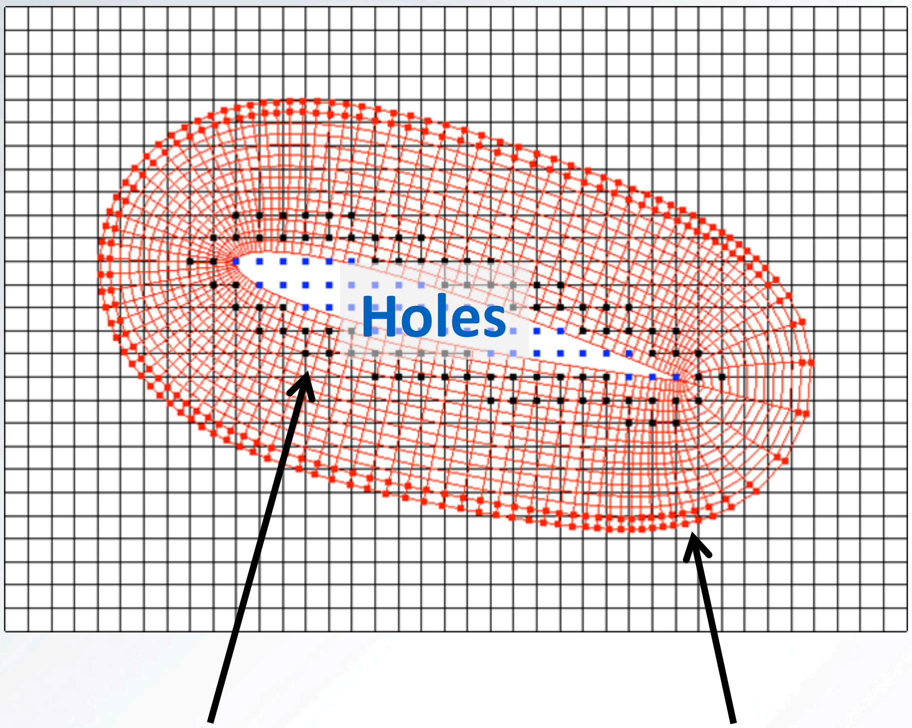
Eliot Quon
Marilyn Smith

Overset Grid Symposium 2014

Georgia Institute of Technology

# Overset State of the Art

**Two additional solution steps required:**




**Fringe points (background)**

**Fringe points (near-body grid)**

1. Domain decomposition
   - Hole cutting
   - Fringe point identification
   - Donor-receptor search
     → **can result in "orphan" points**
2. Data transfer
   - **Interpolation** from *donor* points to *receptor* points
   - Typically trilinear interpolation or linear mapping applied

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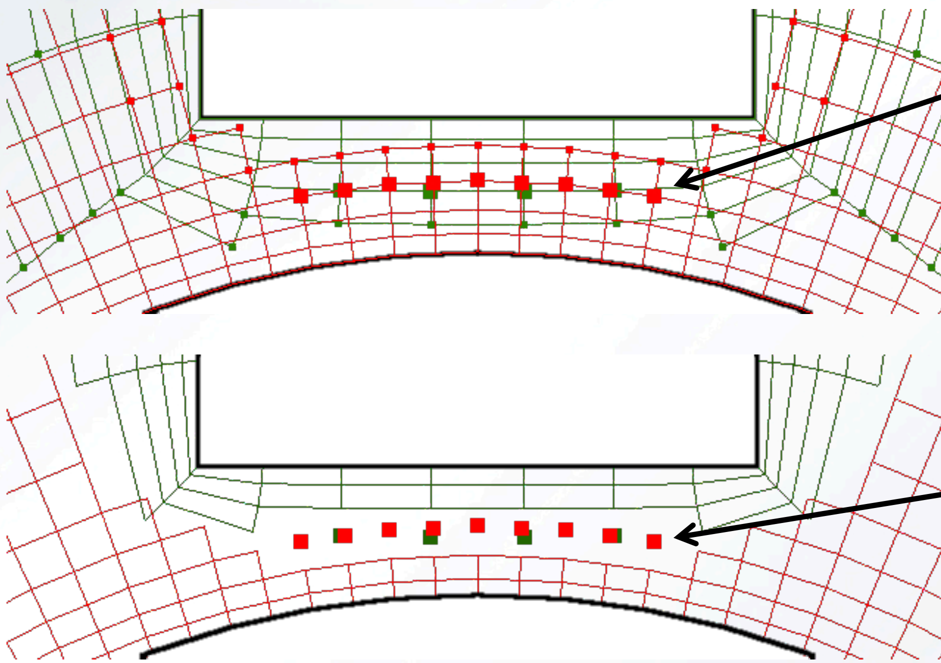
# Overset State of the Art: Orphans

- Two levels of fringes required to maintain high-order accuracy (Gaitonde & Visbal 2000)
- Orphan points can arise due to inadequate grid overlap (depicted by square symbols)

Fringes shown (points)

Fringes hidden



Orphans due to poor donors, e.g. donors are fringe or out

Gap between meshes → **extrapolation** will be necessary

Source: R.W. Noack and D.A. Boger, "Suggar++ Users Guide for Version 1.0.28", October 1, 2010.

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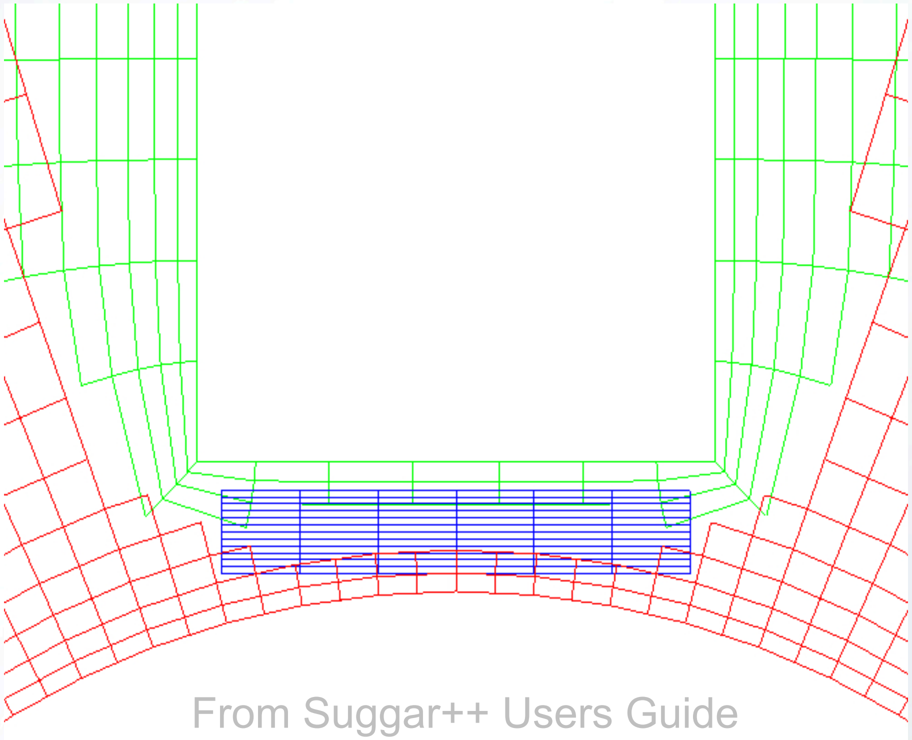
# Treatment of Orphan Points

- Orphans handled by averaging (Rogers et al. 2000, Noack 2006)
- User intervention
  - Interface grid may be added (in blue on the right)
  - Redesign grids for improved overlap
  - Added cost in engineering hours or computation time
- Accept reduced fidelity due to unresolved or poorly resolved fringe points



From Suggar++ Users Guide for Version 1.0.28

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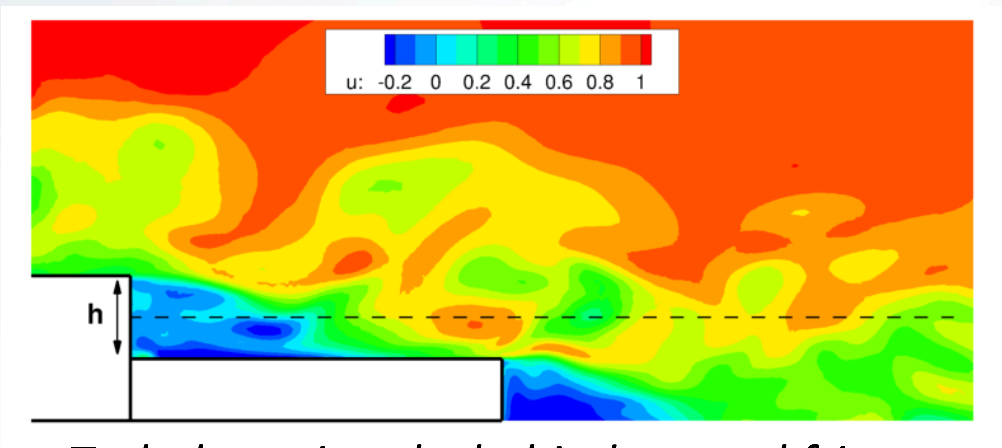
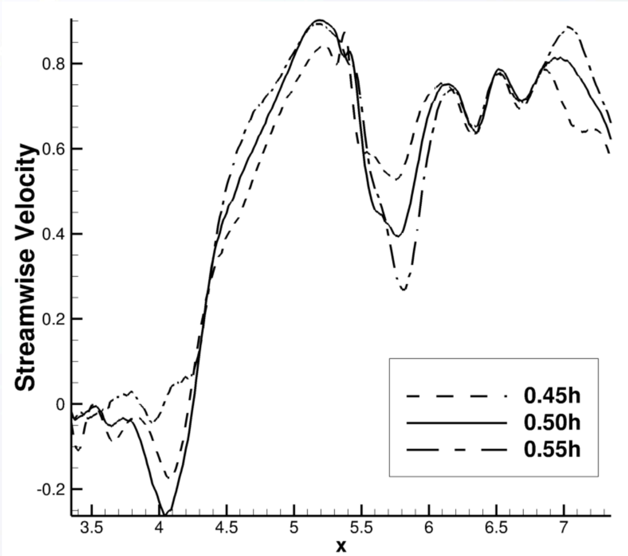
# Data Transfer State of the Art

- **Inconsistency**: Flow solver order of accuracy (≥2) and data transfer accuracy (linear interpolation, order=2) (Chesshire & Henshaw 1990, Delfs 2001)
- Linear overset interpolation techniques can cause **distortion of flow features** (Foster & Noack 2012)
- Degraded solution quality in the vicinity of **fluctuations and discontinuities** (Delfs2001), e.g., acoustic pressure and turbulent flow fields




*Turbulent airwake behind a naval frigate*

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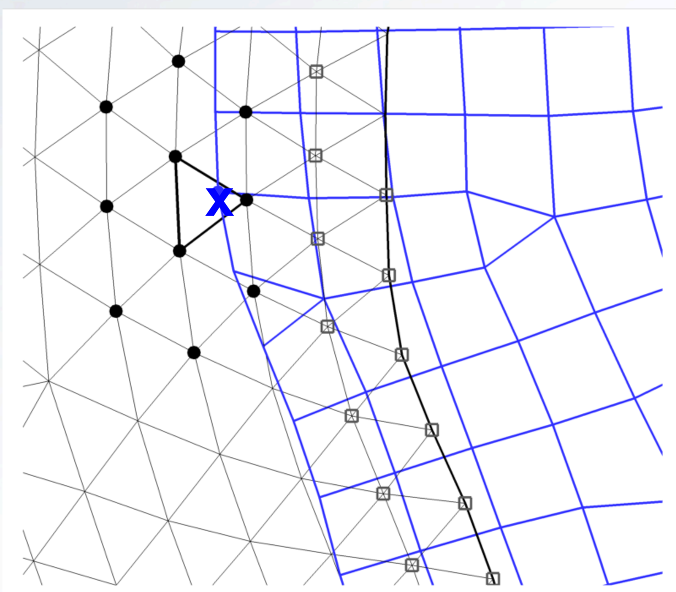
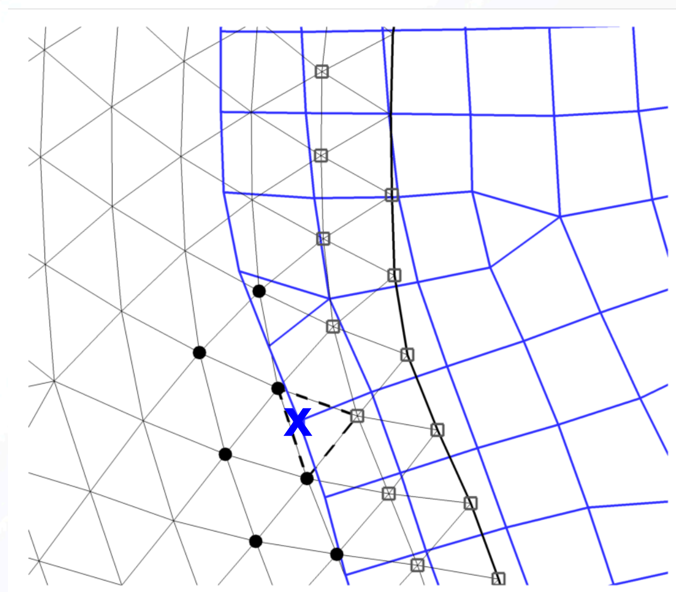
# A Cloud-Based Approach

- Interpolation and extrapolation based on a set of arbitrarily distributed donor points within a neighborhood of a receptor
- Approach is completely stencil free, applicable to any topology in any dimension
- **Handles configurations with/without orphans in the same manner**




**Normal fringe point**




**Orphan point**

**x : receptor**
**● : selected donor pts**
**○ : fringe pts on donor grid**

# Selection of Mapping Technique

- Applying high-order polynomials can be problematic
  - Spurious oscillations possible (Desquenes et al. 2006)
  - Can apply a filtering approach to select coefficients that minimize error (Sherer & Scott 2005) or a limiter for stability (Lee et al. 2011)
  - For arbitrary distributions of points, system of equations not guaranteed to be solvable (Sherer & Scott 2005), limiting applicability for grids with orphans
- Finite element isoparametric mappings retain dependency on connectivity and application as a cloud-based approach susceptible to numerical error
- Focus is therefore on scattered data interpolation techniques with **radial basis functions** (RBFs)

$$\phi(r) = \Phi(\vec{x}, \vec{x}_c) = \Phi(\|\vec{x} - \vec{x}_c\|_2)$$

Basis is function of Euclidean distance between *x* and chosen RBF centers

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# Radial Basis Functions

The interpolant:

$$s_{f,X}(x) = \sum_{j=1}^{N} \alpha_j \Phi(x - x_j) + \sum_{k=1}^{Q} \beta_k p_k(x)$$

subject to:

$$\sum_{j=1}^{N} \alpha_j p_k(x_j) = 0$$

- Can be applied to the **global** set of all grid points or a local subset of points within a **neighborhood** of each receptor
- Solution approach

$$\begin{bmatrix} A_{\Phi,X} & P \\ P^T & 0 \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \begin{Bmatrix} f|X \\ 0 \end{Bmatrix}$$

  1. Form LHS based on chosen basis function Φ(x), optional polynomial p(x), and set of RBF centers X
  2. Solve linear system for interpolation coefficients, α and β
  3. Rewrite LHS based on target points
  4. Solve for the interpolant (RHS)
- Trade-off between accuracy and numerical stability

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# Choice of Radial Basis Function

- Radial basis functions (RBFs) with shape parameters can require optimization for best performance
  - Can apply least squares or Leave-one-out Cross-Validation (LOOCV) approaches (Casa & Krueger 2013)
  - Added uncertainty and cost
- Selected RBFs: Globally-supported **Thin-plate spline** (Smith et al. 2000) and compactly-supported **Wendland $C^2$** (de Boer et al. 2010, Rendall & Allen 2009, Costin & Allen 2013) basis functions considered
  - Compact functions are identically zero outside of a nominal support radius, permitted to vary throughout the flow field depending on local mesh density
  - Compact support can improve performance and conditioning of interpolation problem
  - By definition Wendland $C^2$ is twice continuously differentiable

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# Radial Basis Function Properties

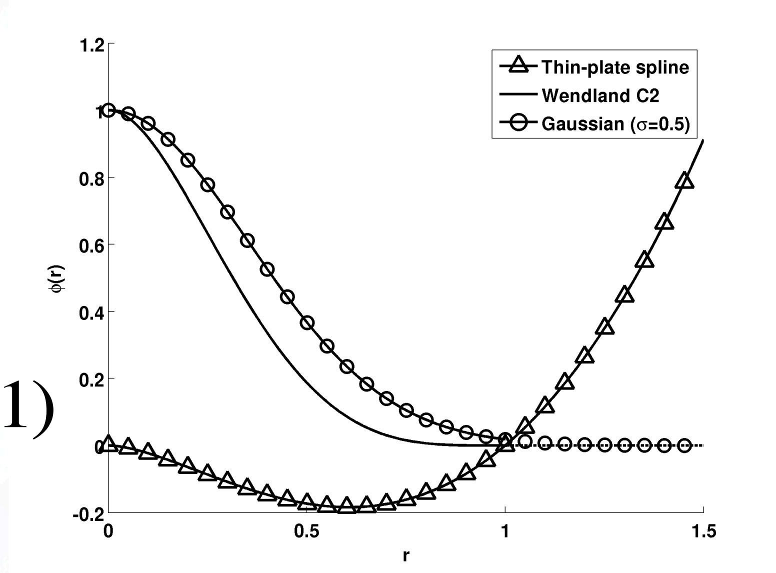
- Choice of donor points coincident with RBF centers guarantees existence of interpolant (Mairhuber-Curtis theorem)
- Positive-definite functions guarantee invertibility of interpolation matrix(Wendland 2005)
- Compact functions (Schaback 1995) are positive definite and may be derived with arbitrary smoothness properties (Wendland 2005)
- Donor points rescaled to lie on unit domain, implicitly specifying the support radius

**Thin-plate spline:**

$$\phi(r) = r^2 \log r$$

**Wendland $C^2$:**

$$\phi(r) = (1-r)^4(4r+1)$$




Compact function identically zero for r>1

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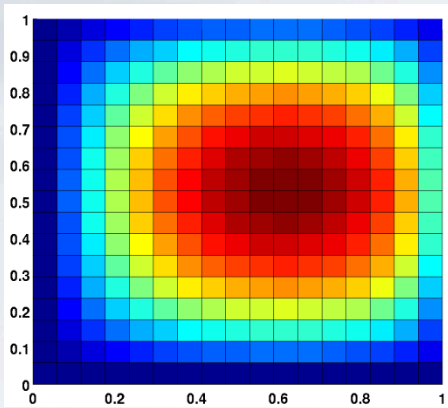
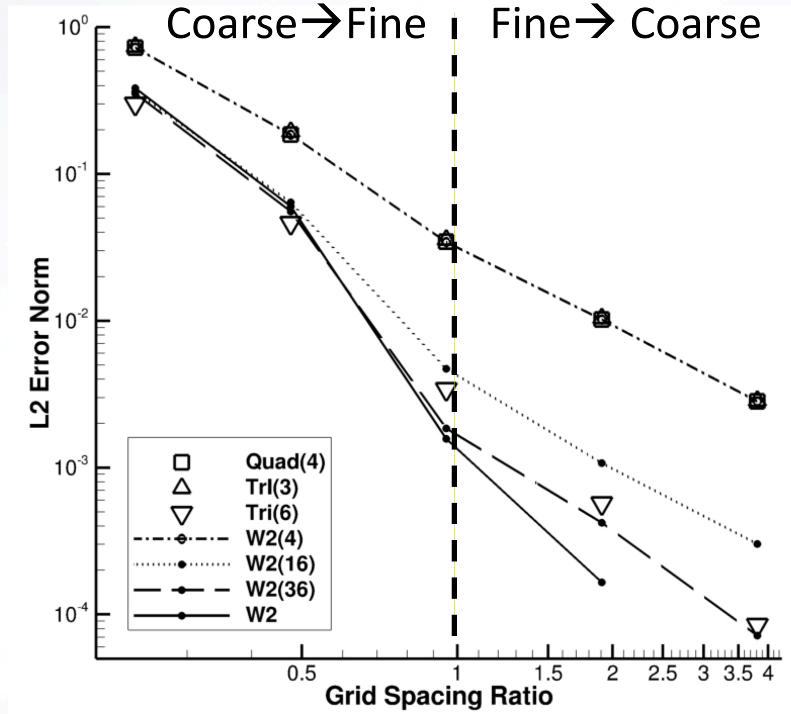
# Interpolation Order of Accuracy

*Sample analytical PDE solution*




- Improvements over linear techniques achieved with more donor points
- RBF accuracy increases with donor points (fine-to-coarse interpolation)
  - With 36 donors, error converges at ~$O(h^{2.5})$
  - In global limit, ~$O(h^{3.4})$
- With fewer donor points or coarse-to-fine interpolation, approximately second order




| | |
|---|---|
| Quad(4) | = bilinear interpolation |
| Tri(3) | = linear mapping with barycentric coordinates |
| Tri(6) | = quadratic mapping |
| W2 | = Wendland $C^2$ RBF |

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# Computational Methodologies

- **NASA FUN3D** solver
  - Solves unsteady Reynolds-averaged Navier-Stokes (URANS) equations, for compressible, viscous flows on unstructured mixed-element meshes
  - Has overset and adaptive grid capabilities
  - $2^{nd}$ order in space and time
- Auxiliary Codes:
  - **SUGGAR++** (Structured, Unstructured, Generalized overset Grid AssembleR)
  - **DiRTlib** (Donor interpolation Receptor Transaction library)

# Modifications to Computational Tools

- SUGGAR++
  - Interpolation donor points identified using existing neighbor search routines
  - Existing connectivity information used for convenience to expand the interpolation cloud
  - Pre-calculation (and inversion, for non-moving grid problems) of interpolation matrices
- DiRTlib
  - Interpolation weights updated at each time step as **weights are no longer dependent on mesh geometry alone**

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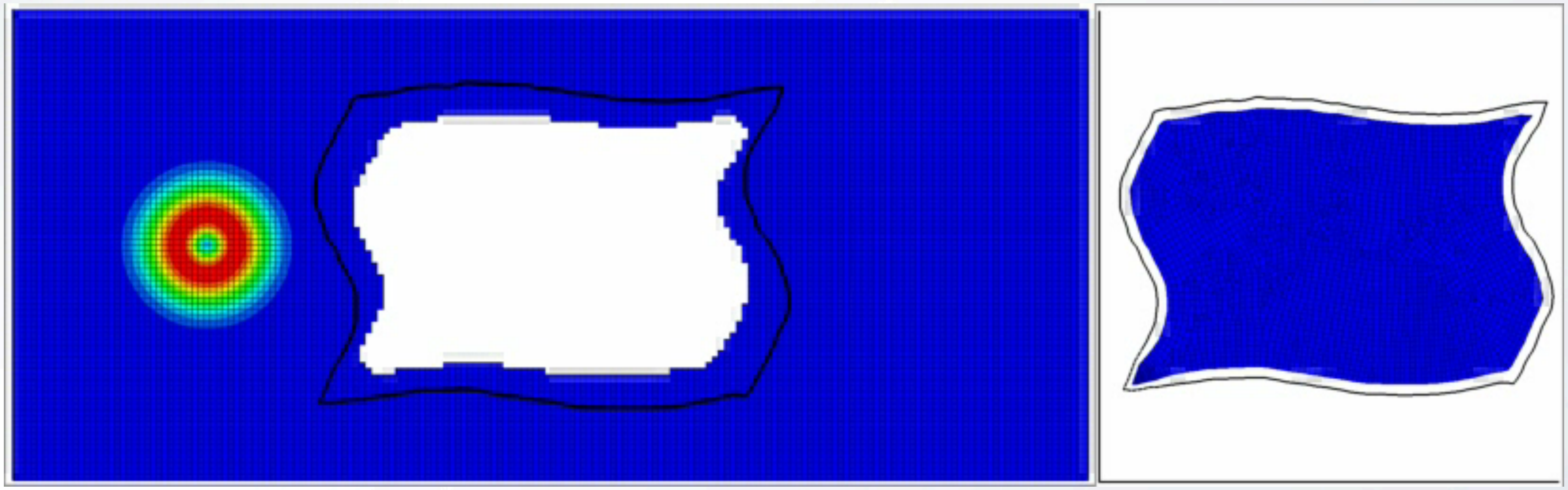
# Convecting Vortex

- Inviscid, incompressible, convecting vortex

$$\frac{u}{U_\infty} = 1 - \frac{C}{U_\infty R}\frac{y - y_c}{R}\exp\left(\frac{-r^2}{2}\right) \qquad \frac{p - p_{norm}}{\rho_\infty U_\infty^2} = 1 - \frac{C^2}{2U_\infty^2 R^2}\exp(-r^2)$$

$$\frac{v}{U_\infty} = \frac{C}{U_\infty R}\frac{x - x_c}{R}\exp\left(\frac{-r^2}{2}\right) \qquad r^2 = \frac{(x - x_c)^2 + (y - y_c)^2}{R^2}$$

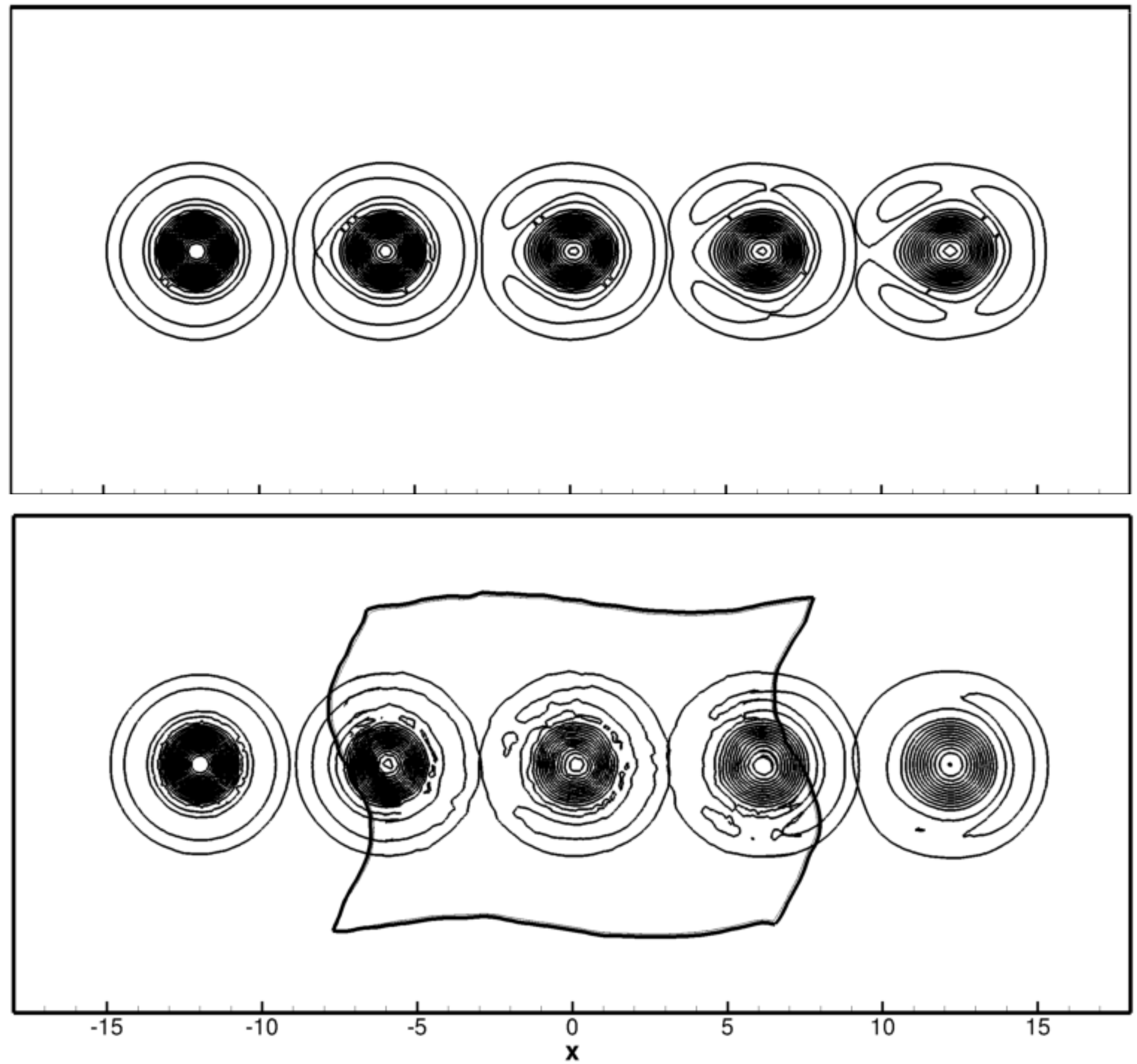
- Unstructured grid system is a simplified version of Sherer & Scott 2005 test case
- Simulation performed using FUN3D and modified auxiliary codes (SUGGAR++, DiRTlib)



# Convecting Vortex

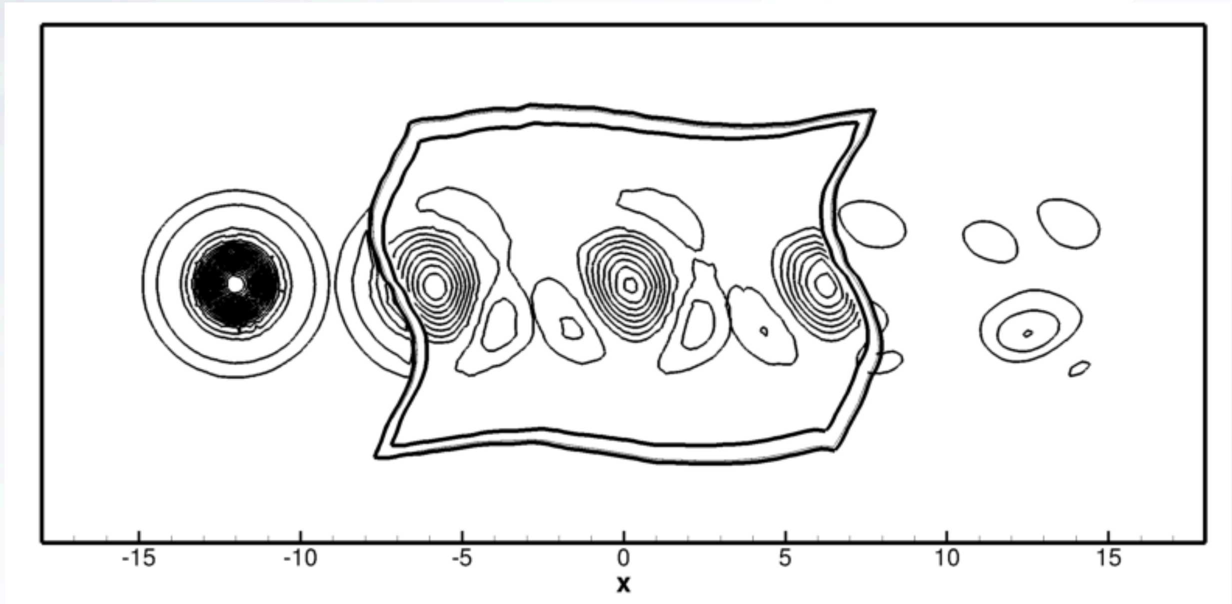
Single grid (reference solution)

**Trilinear interpolation** between overset grids (no orphans)



# Convecting Vortex with Orphan points

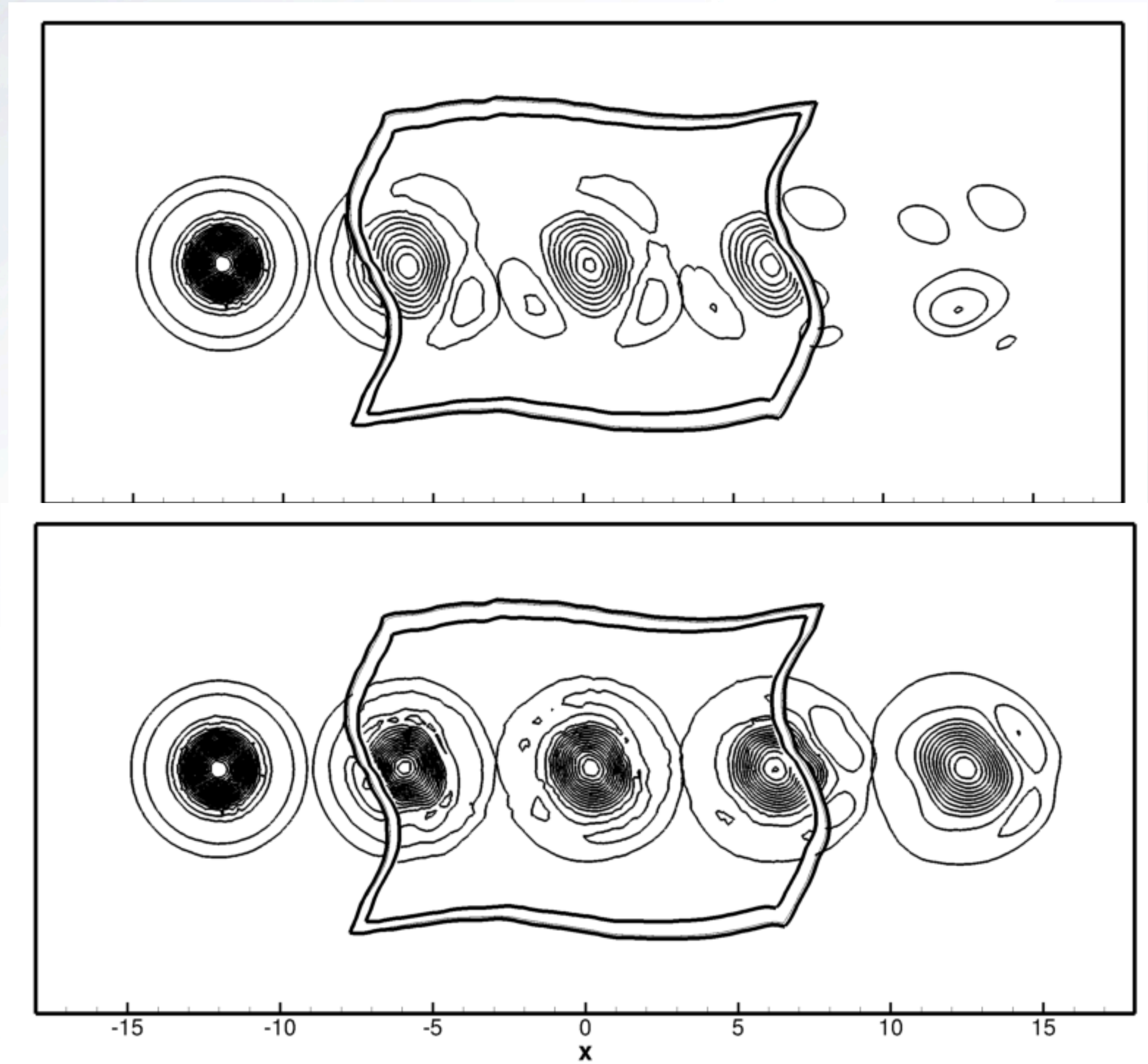
**Trilinear interpolation** between overset grids **with orphans**



# Convecting Vortex with Orphan points

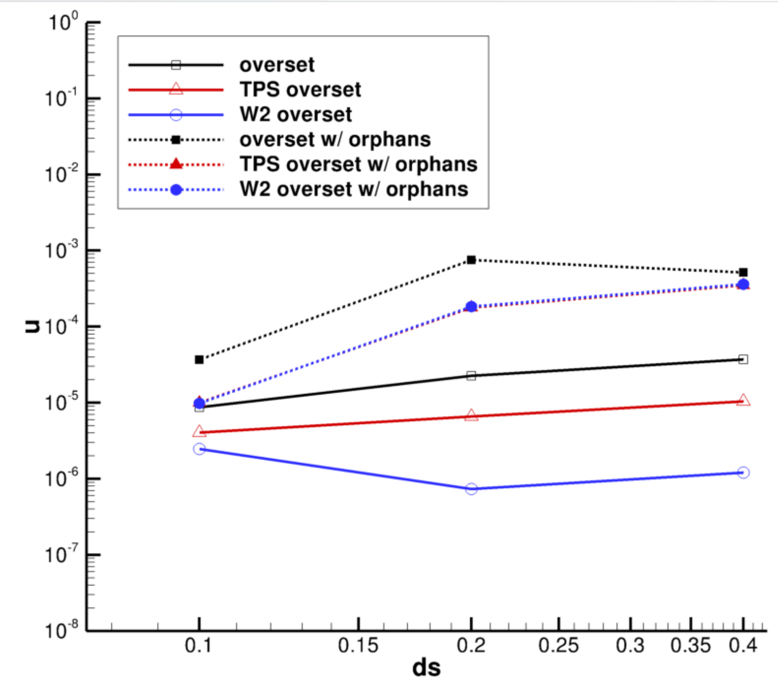
**Trilinear interpolation** between overset grids **with orphans**

**RBF interpolation** between overset grids **with orphans**

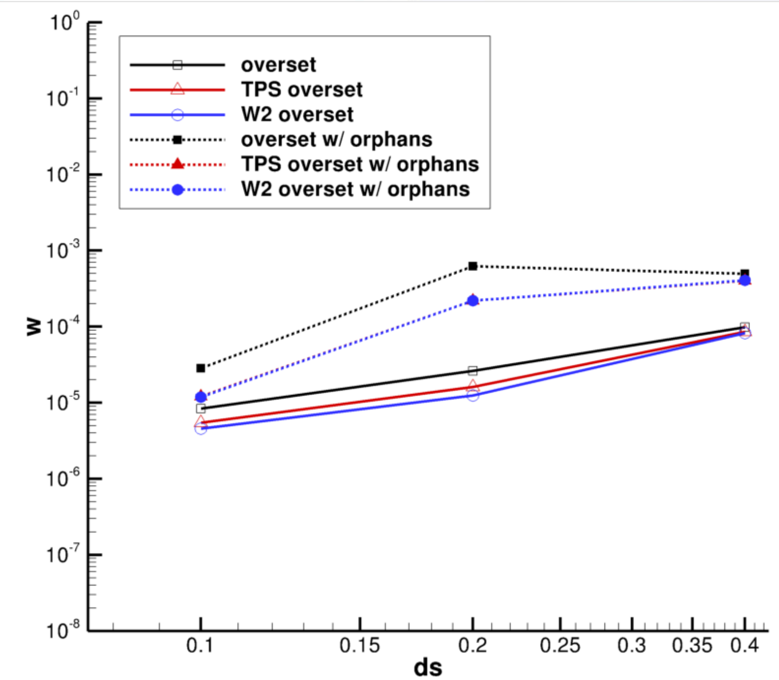


# Convecting Vortex: Isolated Overset Errors

- A time step refinement study was performed to isolate overset errors through Richardson extrapolation
- Overset errors were determined by subtracting the extrapolated single-grid spatial error (0)




Streamwise velocity




Normal velocity

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# Convecting Vortex: Isolated Overset Errors

- Overset error consistently reduced by the RBF overset algorithm for cases both with and without orphans
- At finest grid level, errors are dominated by grid resolution rather than interpolation
- Approximately 30% overset error reduction typically observed without orphans
- Overset errors with orphans typically reduced over 70%

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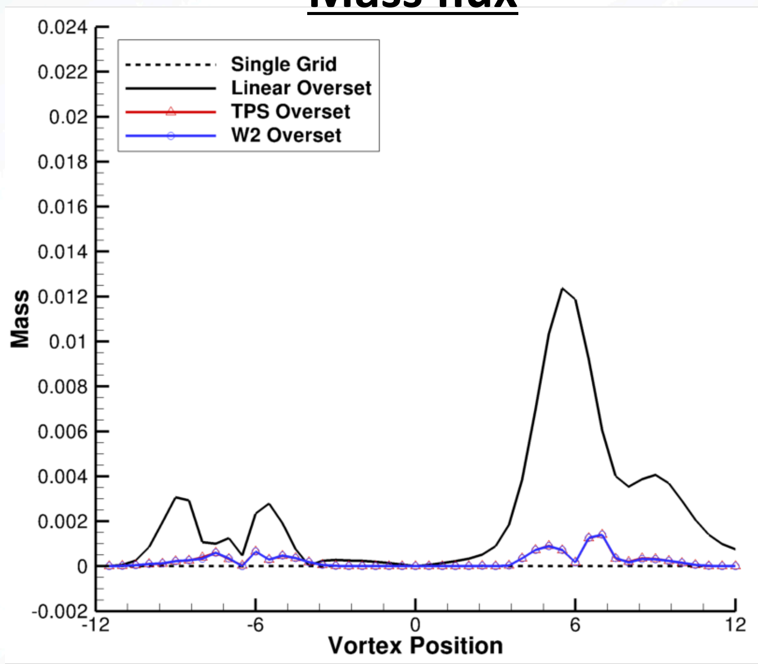
# Convecting Vortex: Conservation Errors

- Mass and momentum fluxes calculated along the outer boundary with Romberg integration quantifying the extent that conservation laws are satisfied when orphans are present
- RBF approaches provide an order of magnitude reduction in transient conservation error

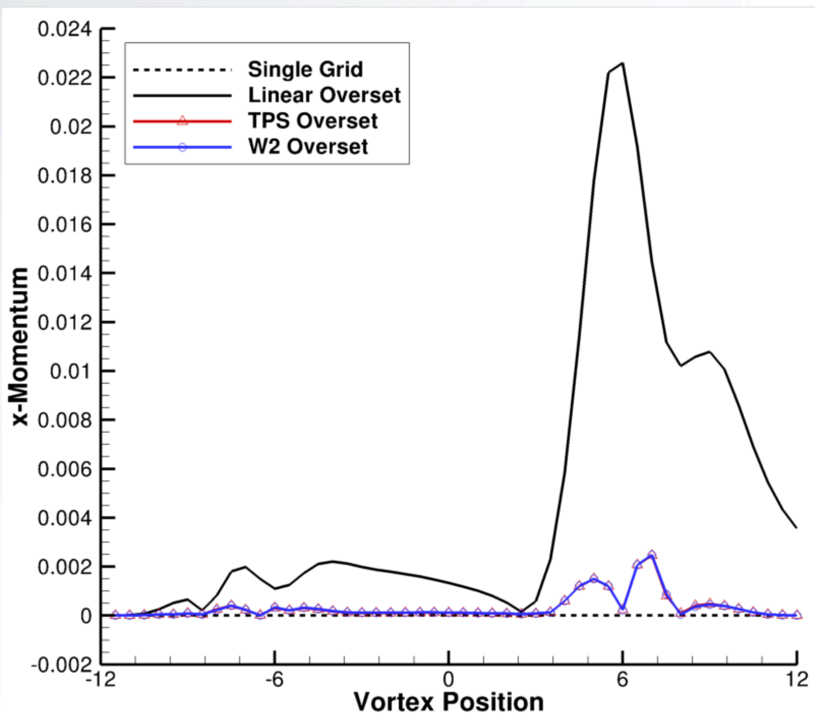
**Mass flux**



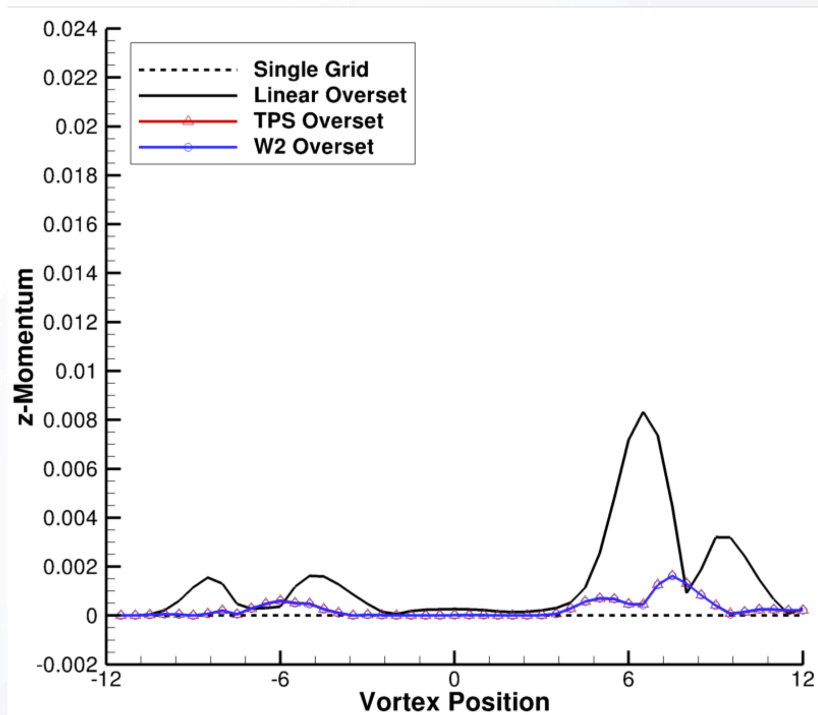
# Convecting Vortex: Conservation Errors

- RBF calculations do not simply scale errors
- Transients persist when applying the traditional linear overset approach

**Streamwise Momentum Flux**

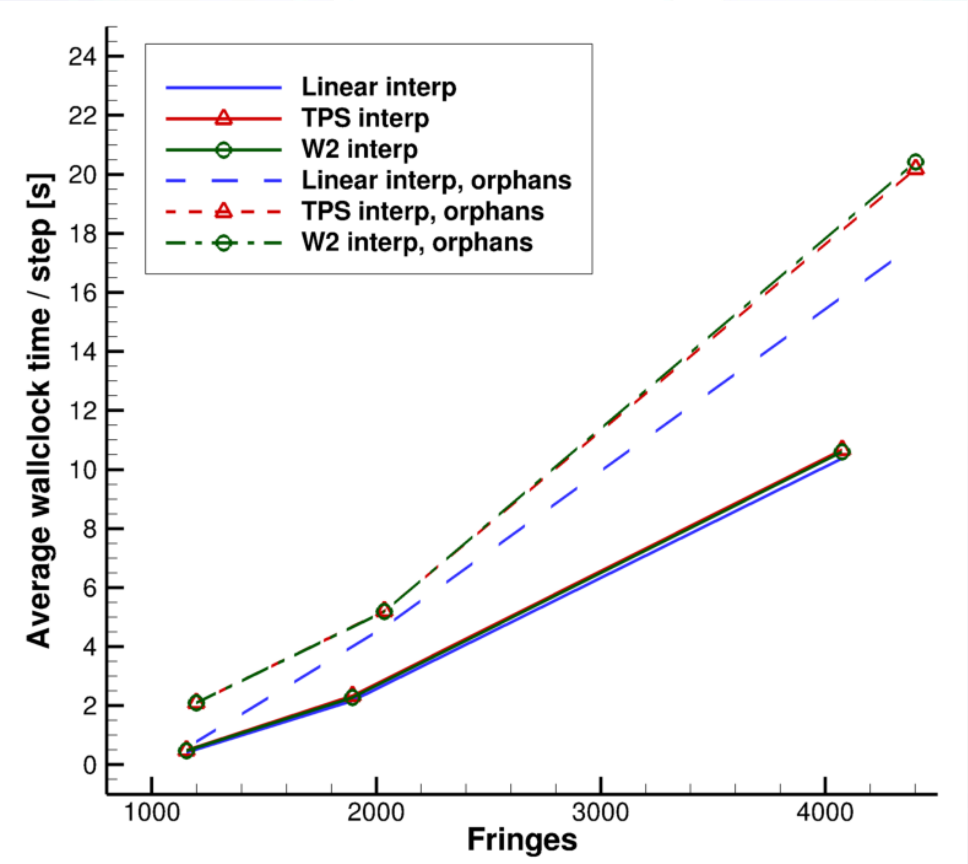



**Normal Momentum Flux**



# Computational Cost

- Cost measured in average wallclock time on baseline/refined grids
- Overset convecting vortex (serial)
  - Minimum cost increase for RBF = 2% (of solver time)
  - Increase in cost ~0.3 s / iteration
- Overset convecting vortex with orphans (serial)
  - 11-16% increase in cost

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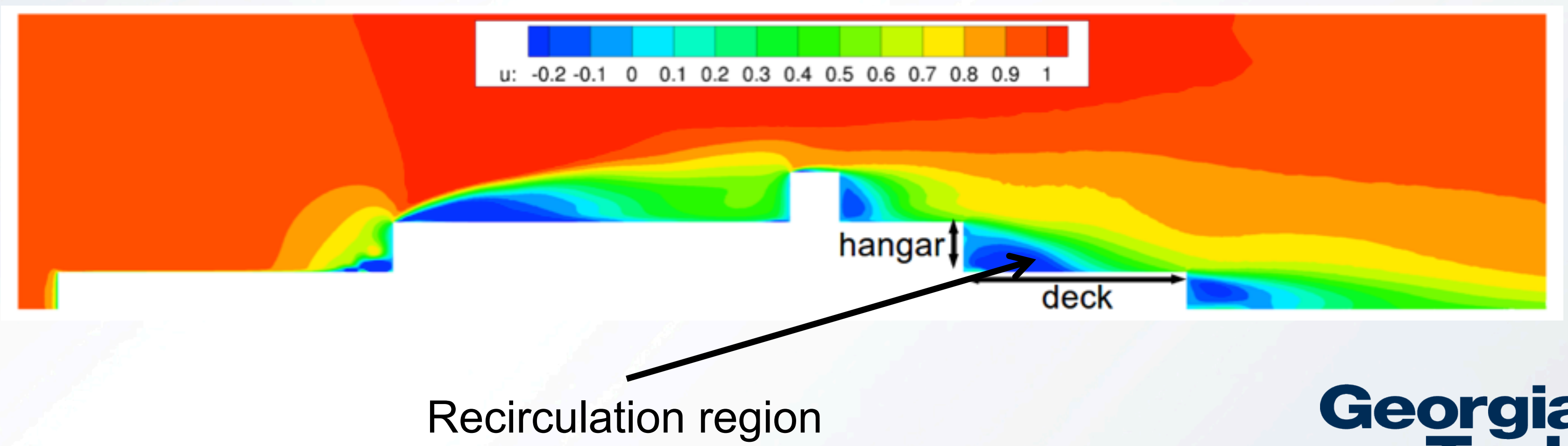
# Turbulent Ship Airwake

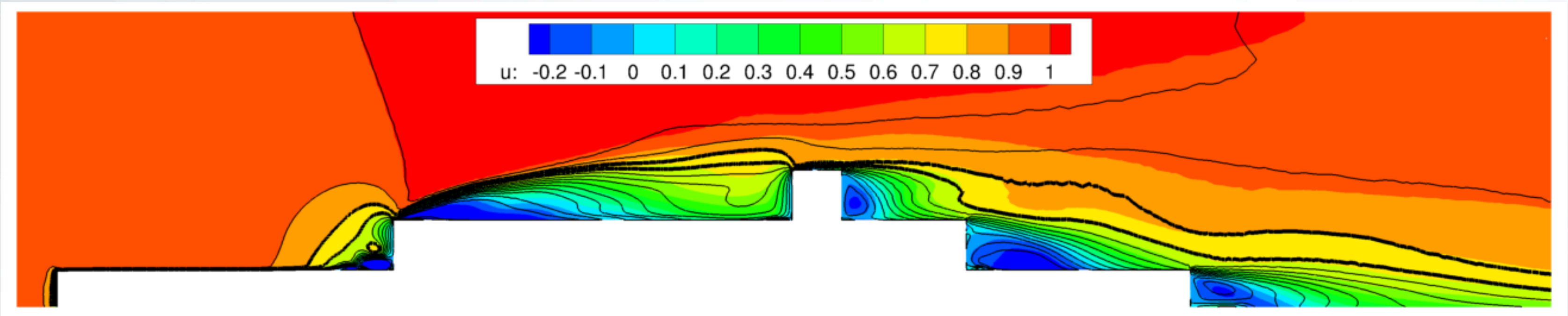
- Headwind flow over a simple frigate shape (SFS2) model
- Near-body ship grid (boundaries extending outward approximately half a ship length) overset onto background grid



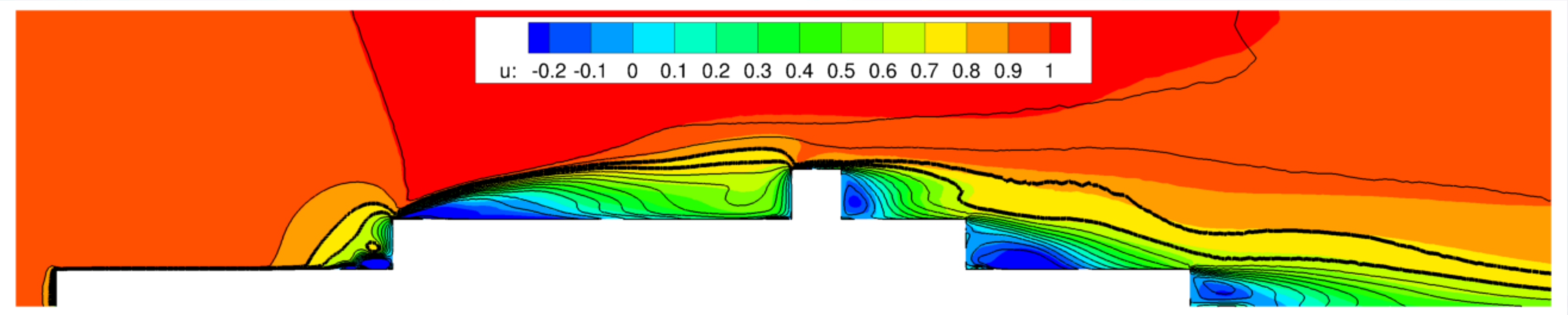
# Turbulent Ship Airwake

- Qualitative differences noted in the hangar wake

**Trilinear overset interpolation**




**RBF overset interpolation**




**Streamlines indicate single grid reference solution**

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# Turbulent Ship Airwake

| Overset Method | Separation Location (normalized by hangar height) |
|---|---|
| Single Grid | 0.525 |
| Trilinear Interpolation | 0.476 |
| Thin-plate spline RBF interpolation | 0.518 |
| Wendland $C^2$ RBF interpolation | 0.517 |

- RBF methods predict separation location on deck to within 2%
- Cost increase per step is approximately 6%, equal to ~1 second per step

# Conclusions

- Cloud-based data transfer with RBFs **eliminates problems with identifying donors for orphan points**
- **Errors consistently reduced** on two- and three-dimensional grids both with and without orphans
  - 0.5-1.5 orders of magnitude reduction on grids with orphans
  - **Transient conservation errors are minimized**
- Added cost of applying RBF interpolation is negligible for normal overset configurations
  - Increase in cost ~11% for cases with orphans on baseline grid
  - Cost can be reduced in parallel (reducing the number of fringes/processor)

# Thank you!

- This research was supported by the Office of Naval Research
- "High-Accurate Physics-Based Wake Simulation Techniques"
- Technical Monitors: Judah Milgram and John Kinzer

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