An Overset Mesh Approach for 3D Mixed Element High Order Discretizations

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Motivation

- High order methods in aerodynamics
- Higher accuracy with fewer degrees of freedom
- Discontinuous Galerkin finite elements
- Use DG with overset grids
  - Nearest neighbor stencil (favorable implications for overset mesh applications)
- Bodies in relative motion
- HELIOS approach: near-body, off-body

Governing Equations

- Compressible Navier-Stokes equations

$$\frac{\partial U_m}{\partial t} + \frac{\partial F_{mi}}{\partial x_i} = 0$$

- Conservative variables $U = \{\rho, \rho u, \rho v, \rho w, \rho E\}^T$

$$F = \left\{ \begin{array}{ccc} \rho u & \rho v & \rho w \\ \rho u^2 + P - \tau_{11} & \rho uv - \tau_{12} & \rho uw - \tau_{13} \\ \rho uv - \tau_{21} & \rho v^2 + P - \tau_{22} & \rho vw - \tau_{23} \\ \rho uw - \tau_{31} & \rho vw - \tau_{32} & \rho w^2 + P - \tau_{33} \\ \rho uH - \tau_{1j} u_j + q_1 & \rho vH - \tau_{2j} u_j + q_2 & \rho wH - \tau_{3j} u_j + q_3 \end{array} \right\}$$

$$\rho E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho(u^2 + v^2 + w^2)$$

DG Formulation

- Multiply by test function and integrate

$$\int_\Omega \phi_r \left(\frac{\partial U_m}{\partial t} + \frac{\partial F_{mi}}{\partial x_i} \right) \mathrm{d}\Omega = \int_\Omega \phi_r S_m \mathrm{d}\Omega$$

- Integrate by parts

$$R_{mr} = \int_\Omega \left(\phi_r \frac{\partial U_m}{\partial t} - \phi_r S_m - \frac{\partial \phi_r}{\partial x_i} F_{mi} \right) \mathrm{d}\Omega + \int_\Gamma \phi_r F^*_{mi} \mathrm{n}_i \mathrm{d}\Gamma = 0$$

- F^*_{mi} is numerical used on interior faces
  - Inviscid flux: Lax-Friedrichs, Roe, and AUFS
  - Viscous flux: symmetric interior penalty (SIP)

DG Solver

- Non-linear system solver: Newton method

$$J^k_{mrns}\Delta a^k_{ns} = \left[\frac{\delta_{mn}M_{rs}}{\Delta\tau} + \frac{\partial R^k_{mr}}{\partial a^k_{ns}}\right]\Delta a^k_{ns} = -R^k_{mr}$$

- Pseudo time step

$$\Delta\tau = \frac{CFL}{h^{-1}(\sqrt{u^2+v^2+w^2}+c)}$$

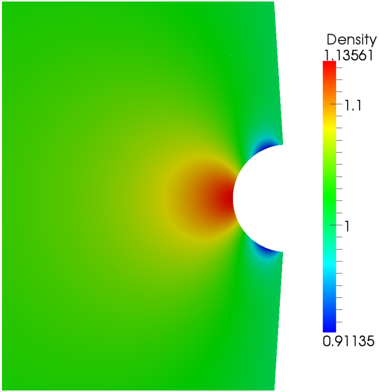
- Linear system solver: preconditioned flexible-GMRES (Saad 1986)
- Line implicit Jacobi, Gauss-Seidel relaxation, ILU(0)
- Full Jacobian or complex Fréchet derivative

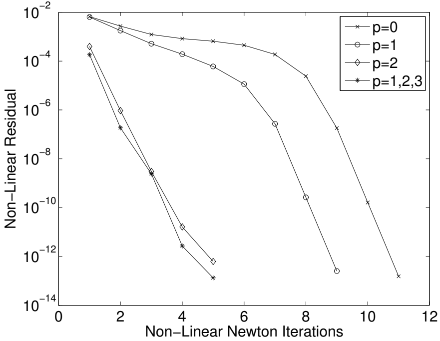
Solver Capabilities

- Hybrid mixed element unstructured meshes (tetrahedra, prisms, pyramids, and hexahedra)
- Polynomial degree up to $p = 8$
- p-enrichment and h-refinement using non-conforming elements (hanging nodes)
- Curved elements
- Independent polynomial degree for solution and mapping basis
- Fully parallelized using MPI
- PDE-based Artificial Viscosity
- Spalart-Allmaras turbulence model (negative-SA variant)

Results: Inviscid Cylinder

- $M_\infty = 0.5$
- *p*-adaption





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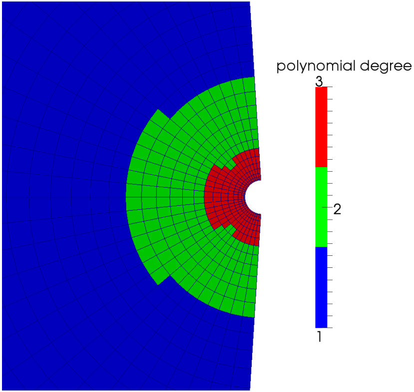
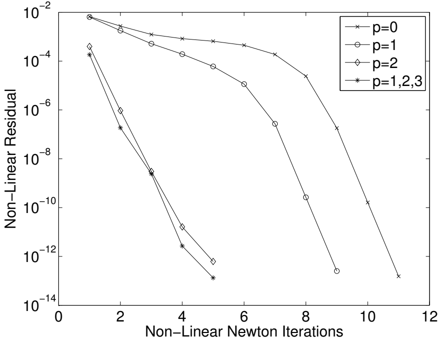
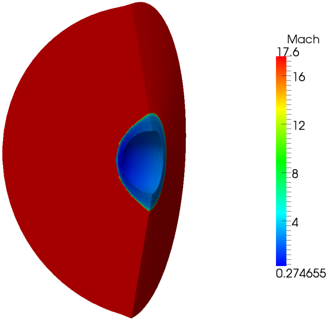
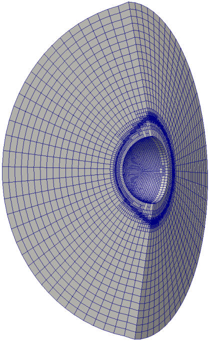




Results: Inviscid sphere

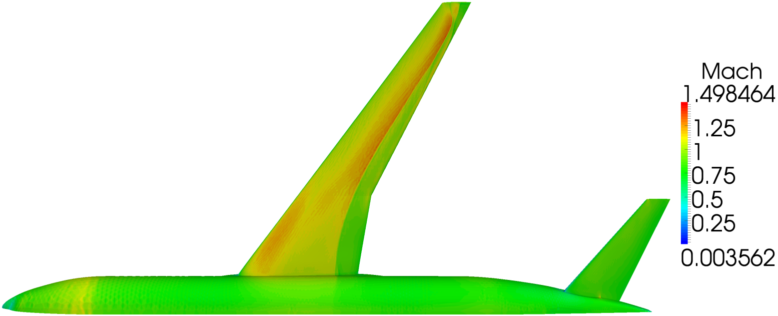
- $M_\infty = 17.6$
- *h*-adaption





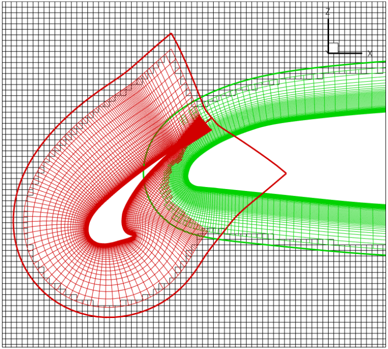
Results: Inviscid CRM

- NASA Common Research Model $M_\infty = .85$, $\alpha = 1^\circ$
- 1.9 million tetrahedra $p = 2$
- 19 million DOF
- Solved on 4096 processors
- Artificial viscosity



High Order Overset

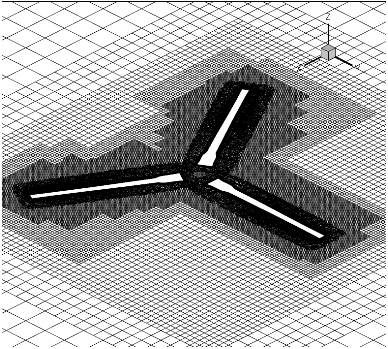
- TIOGA Topology Independent Overset Grid Assembler
  - Developed by Jay Sitaraman
  - Implicit hole cutting strategy
  - Alternating Digital Tree for fast search
  - Fully Parallel
  - Hybrid mixed element
  - Single component grid partition per processor
  - Modified with high order call back functions



(Jay Sitaraman)

High Order Overset Motivation

- Bodies in relative motion
  - Wind turbines
  - Rotorcraft
- Helios strategy with high order accuracy
- Modal Hybrid element DG near-body (tetrahedra, pyramids, prisms, hexahedra)
- Nodal Tensor Product Cartesian DG off-body (Andrew Kirby)



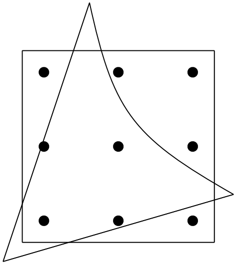
(Jay Sitaraman)

Initialize TIOGA

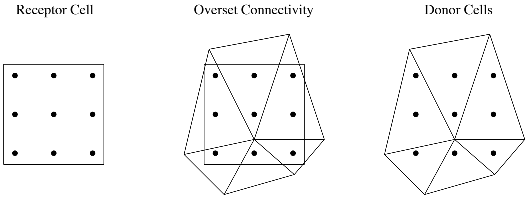
- Register grid data (nodes and connectivity)
- Set high order call back functions
- Perform overset connectivity
- Node based iblanking
- Convert to cell based iblank
- Remove extra fringe cells (only need nearest neighbors)
- Perform high order connectivity
  - Search for donors cells using high order inclusion and find high order interpolation weights
  - High order accurate interpolation (equivalent to solver discretization accuracy)

High Order Call Back Functions

- Subroutines written in Fortran DG code but called from within C++ TIOGA library
- Call back functions:
  - Create receptor nodes on cell
  - Inclusion test (curved cells)
  - Interpolation weights from donor cell
  - Update solution using Vandermode or Mass matrix



Receptor and Donor Cells

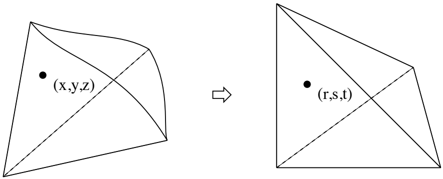


- Create receptor nodes on high order receptor cell
  - Quadrature points or equidistant points
- TIOGA performs overset connectivity
- TIOGA finds possible donor cells through ADT search
- High order call back function tests for inclusion of receptor node in high order donor cells

Inclusion test

- Given physical coordinates (x, y, z) on iso-parametric cell
- Search for natural coordinates (r, s, t) in donor cells
- Natural coordinates are on standard straight sided element
- ϕ are the element basis functions and a,b,c are mapping coefficients for spatial coordinates
- m is total modes

$$\sum_{i=1}^{m} \phi_i(r,s,t)a_i = x \qquad \sum_{i=1}^{m} \phi_i(r,s,t)b_i = y \qquad \sum_{i=1}^{m} \phi_i(r,s,t)c_i = z$$

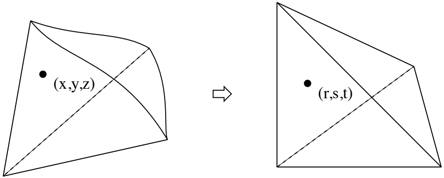


Inclusion test

- Use Newton-Raphson to solve for natural coordinates (r, s, t)

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial s} & \frac{\partial z}{\partial t} \end{bmatrix} \begin{bmatrix} \delta r \\ \delta s \\ \delta t \end{bmatrix} = - \begin{bmatrix} \sum_{i=1}^{m} \phi_i(r, s, t) a_i - x \\ \sum_{i=1}^{m} \phi_i(r, s, t) b_i - y \\ \sum_{i=1}^{m} \phi_i(r, s, t) c_i - z \end{bmatrix}$$

- When natural coordinates are found then determine if within bounds on standard element



$$r + 1 < -\epsilon$$

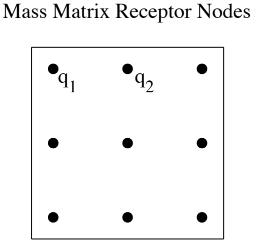
$$s + 1 < -\epsilon$$

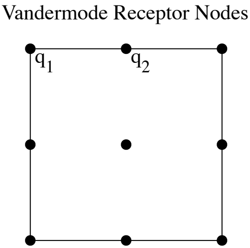
$$t + 1 < -\epsilon$$

$$r + s + t > \epsilon$$

Solution Reconstruction

- Reconstruct high order polynomial representation of solution by using point values (q_k) from TIOGA
- Two methods: Mass matrix and Vandermode

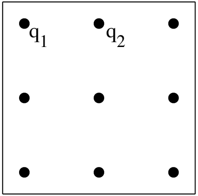




Solution Reconstruction

- Mass matrix solution Reconstruction
- m = total modes
- n = total quadrature points
- For curved mesh $n > m$

Mass Matrix Receptor Nodes



$$\sum_{j=1}^{m} \phi_j(\vec{\xi}_k) a_j = q(\vec{\xi}_k)$$

$$\sum_{j=1}^{m} \int_\Omega \phi_i \phi_j a_j \, \mathrm{d}\Omega = \int_\Omega \phi_i q \mathrm{d}\Omega$$

$$\text{for } i = 1, ..., m$$

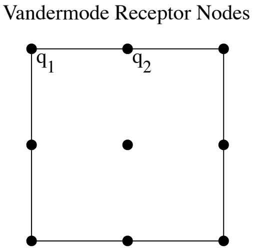
$$M_{ij} = \int_\Omega \phi_i \phi_j \mathrm{d}\Omega$$

$$b_i = \int_\Omega \phi_i q \mathrm{d}\Omega = \sum_{k=1}^{n} \phi_i(\vec{\xi}_k) q(\vec{\xi}_k) w_k$$

$$a = M^{-1} b$$

Solution Reconstruction

- Vandermode solution Reconstruction
- $m =$ total modes
- Number of nodes always equal to number of modes



$$\sum_{i=1}^{m} \phi_i \left(\vec{\xi}_j\right) a_i = q\left(\vec{\xi}_j\right)$$

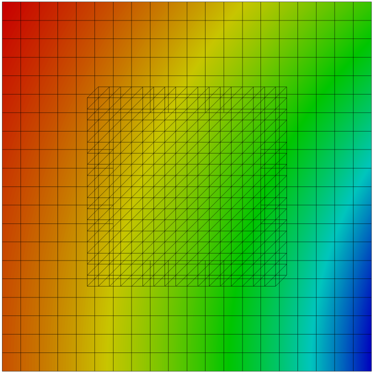
for $j = 1, ..., m$

$$V_{ij} = \phi_i \left(\vec{\xi}_j\right)$$

$$a = V^{-1} q$$

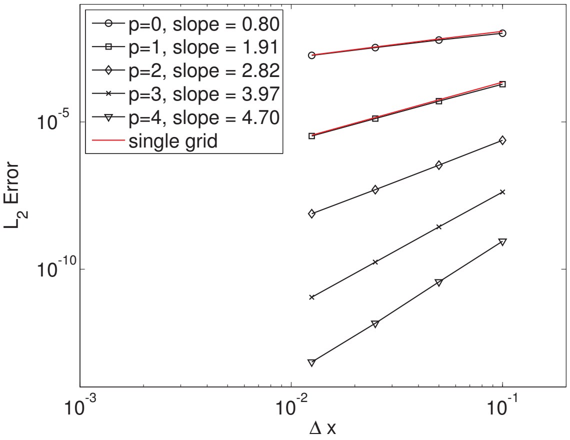
Mesh Resolution Study

- Ringleb: exact solution to 2D Euler equations
- Hexahedral background grid and refined prismatic grid inside
- Refine by 2 each element and measure error compared to exact solution
- Contours of pressure

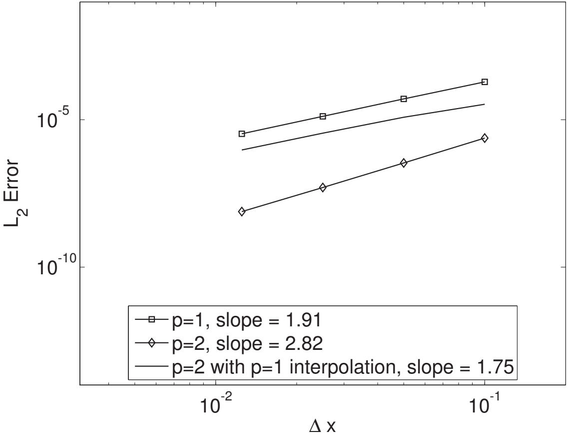


Error vs Mesh Size

- Error decreases with $C\Delta x^{p+1}$
- Single grid results for $p = 0, 1$ match closely to overset

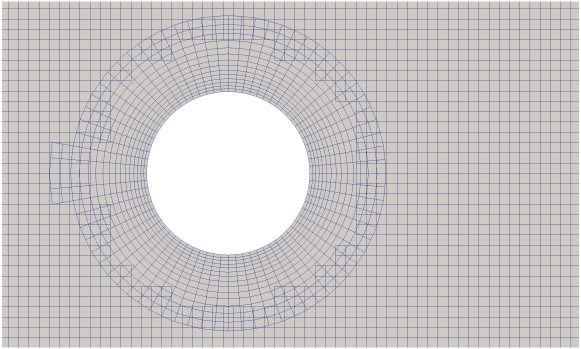


Error vs Mesh size



- Order consistent interpolation gives design accuracy

Overset Cylinder

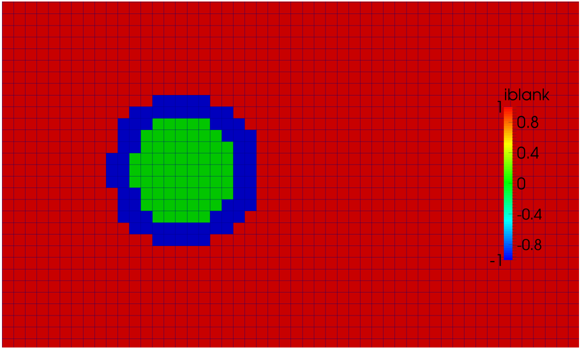


- Curved structured hexahedral grid around cylinder
- Cartesian hexahedral grid in background
- Field cells plotted

Reduce Fringe

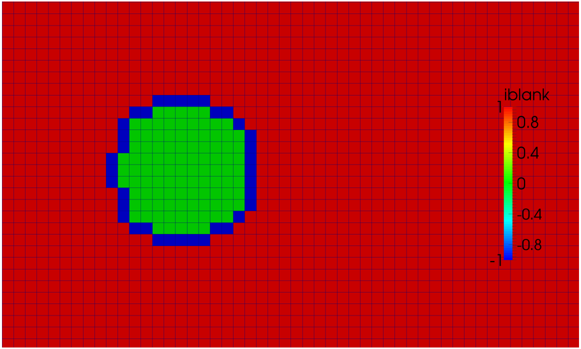
- DG only requires nearest neighbor
- TIOGA returns extra fringe cells
- Minimize fringe to minimize interpolation between grids
- Remove fringe Algorithm:
  - Set all iblank to holes
  - Loop through faces and check left and right cells iblank
  - If field next to field or fringe then set iblank to original value
  - If fringe is next to fringe then do nothing
  - If fringe is next to hole then do nothing

Overset Cylinder



- Cell iblanking: 1=field (red), 0=hole (green), -1=fringe (blue)
- DG only requires nearest neighbors
- Remove fringe cells and convert to hole cells

Overset Cylinder



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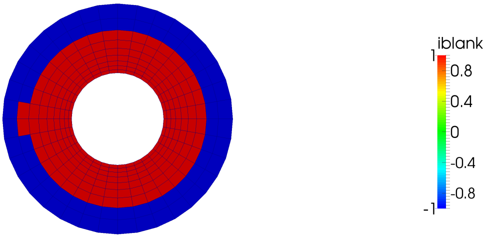

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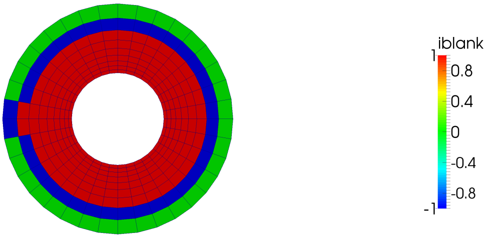

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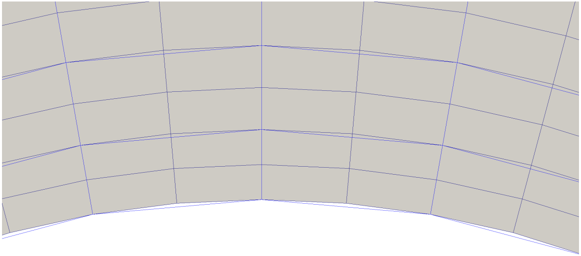
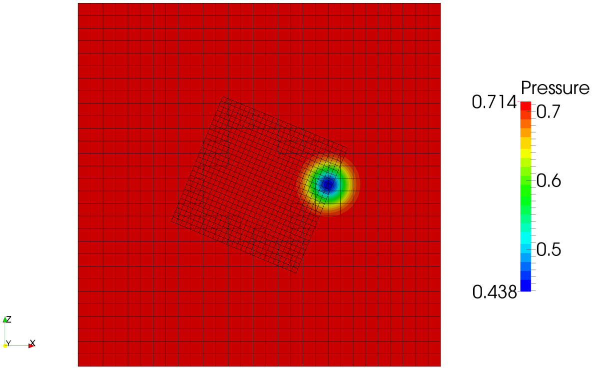
Overset Cylinder

- Cylinder surface is curved using analytic function
- Curvature is pushed radially outward along lines
- Working on generalized CAD mesh curving for complex geometry



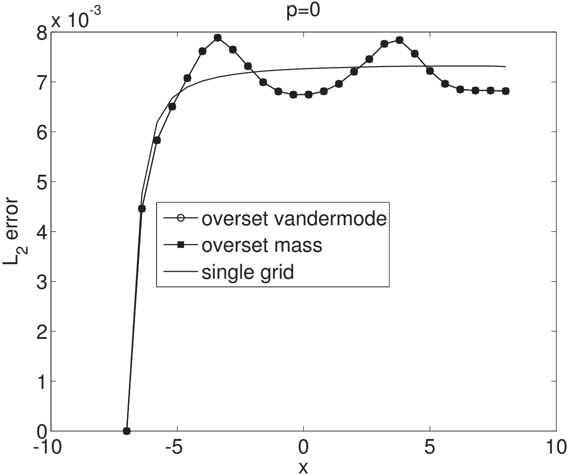
Isentropic Vortex Movie



- Isentropic inviscid vortex advected through domain
- Hexahedral background grid and rotated hexahedral grid

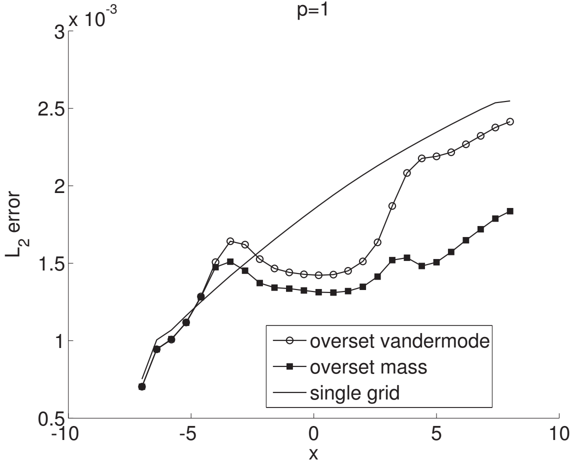
Isentropic Vortex

- Vandermode and Mass Matrix interpolation strategies give nearly identical results
- Increased error in overset region
- decreased error in refined region
- overset outperforms single grid



Isentropic Vortex

- Mass more accurate than Vandermode
- Increased error in overset region
- decreased error in refined region
- overset outperforms single grid



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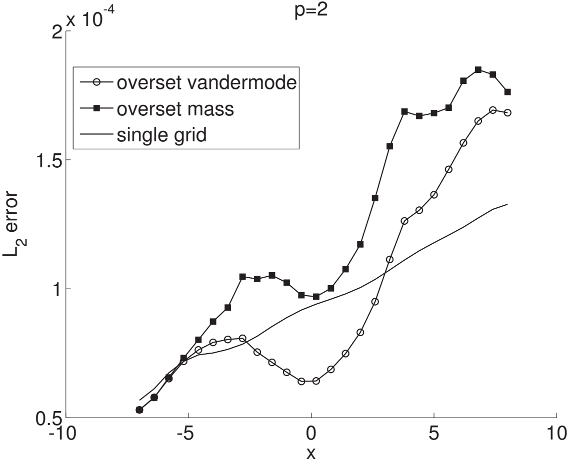

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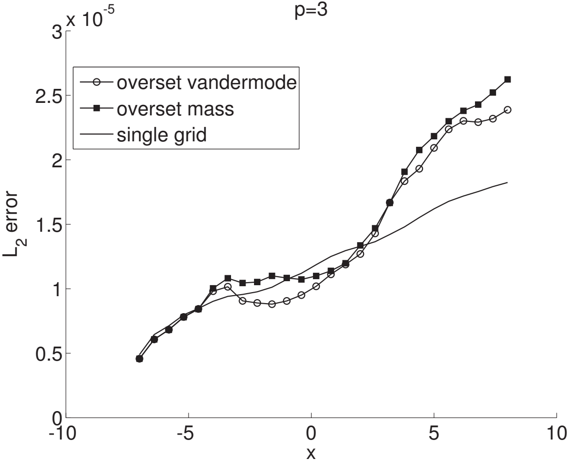
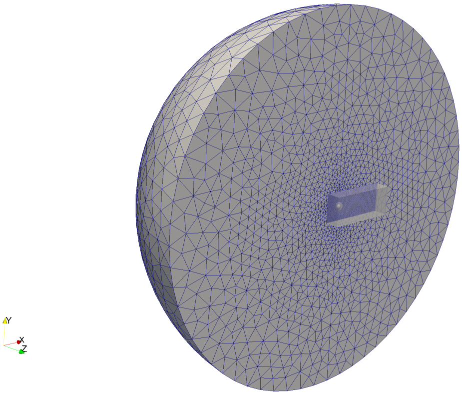


Overset Viscous Sphere

- Strand grid on sphere prisms (curved)
- Hexahedral wake grid
- Tetrahedral off body background grid



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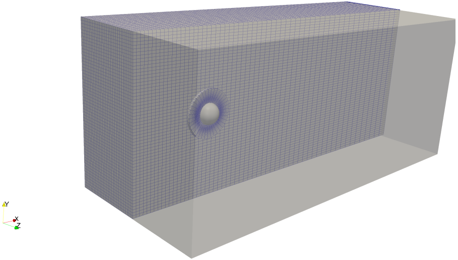
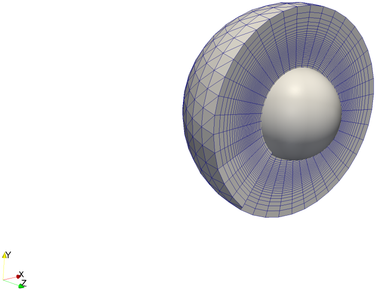
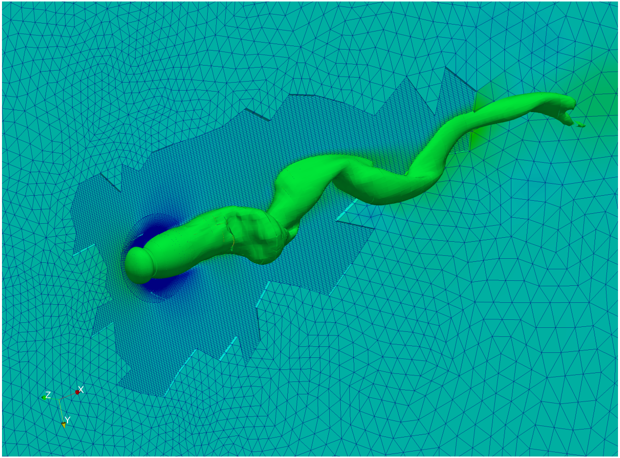




- Strand grid on sphere prisms (curved)
- Hexahedral wake grid

Overset Viscous Sphere

- $p = 2$
- $Re = 1000$
- $M = 0.3$
- 6.8×10^6 DOF
- Contours of temperature



Conclusions

- Developed 3D parallel overset method for high order DG discretizations
  - High order call back functions
  - Curved elements
  - Order preserving interpolation
  - Requires only nearest neighbor stencil
- Future work
  - Combine with off-body cartesian DG solver
  - Large scale RANS and/or LES simulations