A General Implicit Artificial Boundary Scheme for Chimera Methods

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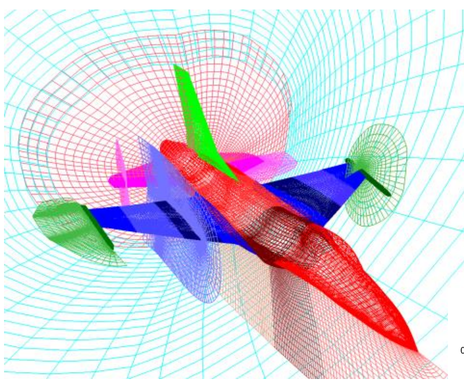
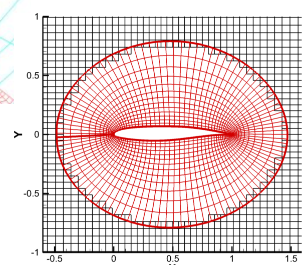
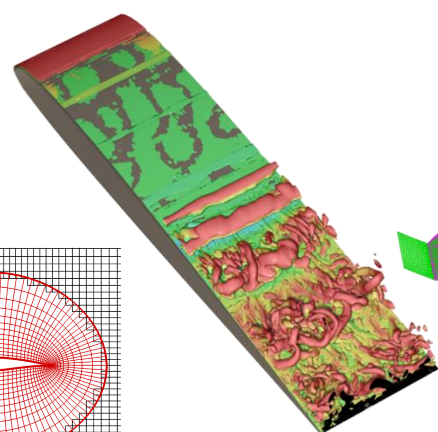
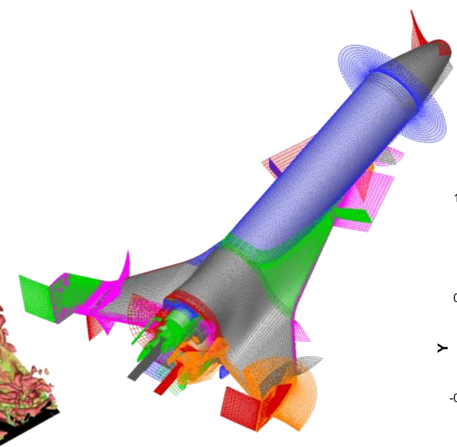
Motivation

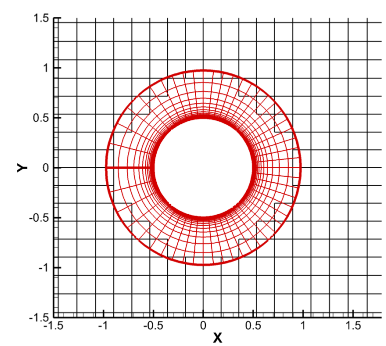
- Chimera Overset Grid Method
  - Complex Geometries
  - “Hot swap” Geometric Features
  - Moving Grids with Relative Motion
    - Store Separation
    - Rotorcraft
- Explicit Artificial Boundaries
  - Solve Decoupled System
  - Limits CFL Number with Increasing Number of Processor
- Implicit Artificial Boundaries
  - Significant Increased Parallel Performance
  - It’s easier than it sounds





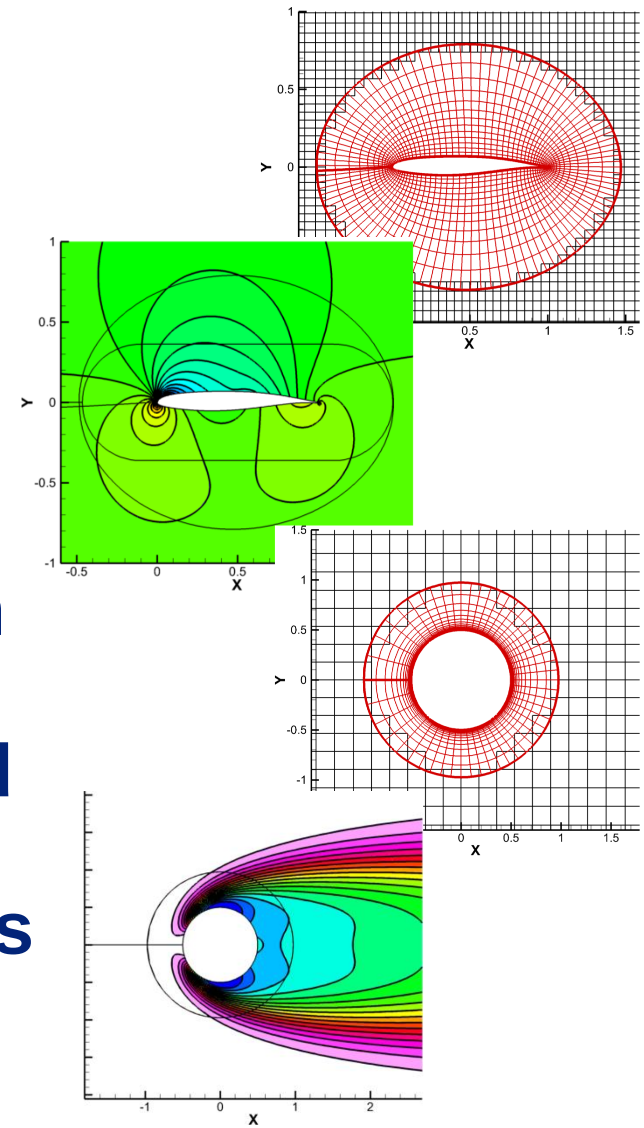






Outline

- **Discretization Assumptions**
- **Explicit/Implicit Chimera**
- **Sparse Iterative Solvers**
  - **Preconditioners**
- **Distributed Memory Parallelism**
- **Discontinuous Galerkin Method**
- **Inviscid/Viscous Flow Examples**
- **Conclusion and Future Work**



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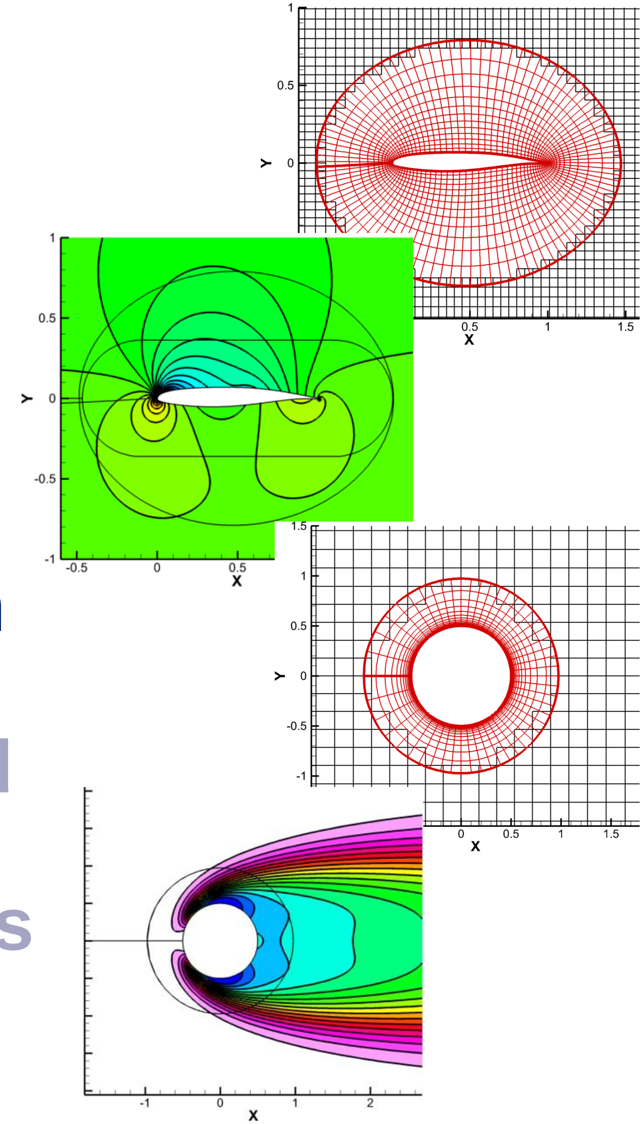
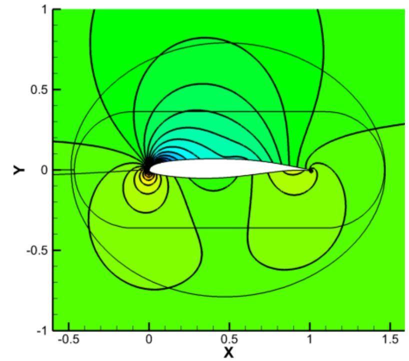


Discretization Assumptions

- **Euler/Navier-Stokes Equations in Conservation Form**

$$\nabla \cdot \vec{F}(Q) = 0$$



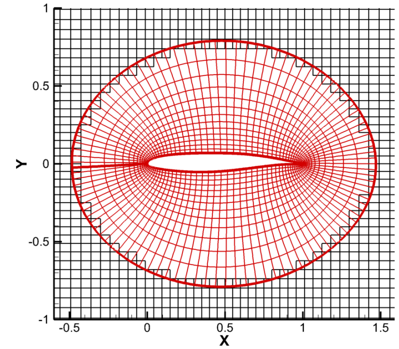
- **Discrete Form**
  - Finite Difference
  - Finite Volume
  - Finite Element

$$R(Q) = 0$$

- **Newton's Method**

$$\frac{\partial R(Q)}{\partial Q}\Delta Q = -R(Q) \qquad A\Delta Q = -R(Q)$$

- **Chimera Interpolation Operator**
  - Linear Operator (don't think 2nd order accuracy)
  - Polynomial Basis Functions
  - Radial Basis Functions
  - Trigonometric Basis Functions
  - etc.



$$I_h(Q) = \sum Q_i \varphi_i \qquad \frac{\partial I_h(Q)}{\partial Q}\Delta Q = I_h(\Delta Q) = \sum \Delta Q_i \varphi_i$$

Explicit/Implicit Artificial Boundaries

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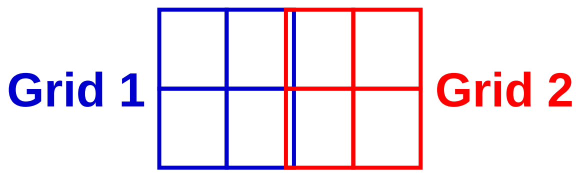
Newton's Method

$$A\Delta Q = R$$



$$R^1\left(Q^1, I_h\left(Q^2\right)\right)=0 \qquad R^2\left(Q^2, I_h\left(Q^1\right)\right)=0$$

- **Unstructured A Matrix**
  - Explicitly add C^i Matrices
- **Structured A^i Matrix**
  - Tri-, Penta-, Hepta-diagonal
- **Sparse Iterative Solver**
  - No Explicit C^i Matrices

Explicit Artificial Boundary

$$\begin{bmatrix} A^1 & 0 \\ 0 & A^2 \end{bmatrix} \begin{bmatrix} \Delta Q^1 \\ \Delta Q^2 \end{bmatrix} = \begin{bmatrix} R^1\left(Q^1, I_h\left(Q^2\right)\right) \\ R^2\left(Q^2, I_h\left(Q^1\right)\right) \end{bmatrix}$$

$$A^1 = \frac{\partial R^1}{\partial Q^1} \qquad A^2 = \frac{\partial R^2}{\partial Q^2}$$

Solve Decoupled System

$$A^1 \Delta Q^1 = R^1\left(Q^1, I_h\left(Q^2\right)\right)$$

$$A^2 \Delta Q^2 = R^2\left(Q^2, I_h\left(Q^1\right)\right)$$

Implicit Artificial Boundary

$$\begin{bmatrix} A^1 & C^1 \\ C^2 & A^2 \end{bmatrix} \begin{bmatrix} \Delta Q^1 \\ \Delta Q^2 \end{bmatrix} = \begin{bmatrix} R^1\left(Q^1, I_h\left(Q^2\right)\right) \\ R^2\left(Q^2, I_h\left(Q^1\right)\right) \end{bmatrix}$$

$$C^1 = \frac{\partial R^1}{\partial Q^2} \qquad C^2 = \frac{\partial R^2}{\partial Q^1}$$

Sparse Iterative Solvers

- Iterative methods for sparse linear systems
  - http://www-users.cs.umn.edu/~saad/books.html

- Restarted GMRES
  - Simple Fortran Code Available
  - http://people.sc.fsu.edu/~jburkardt/f_src/mgmres/mgmres.html

- Fundamental Operations
  - Dot products
  - Sparse Matrix-Vector Multiplication

- Slow without Preconditioner

Sparse Iterative Solvers Preconditioners

- Incomplete LU
- ARC3D Beam-Warming block tridiagonal scheme.
- F3D Steger-Warming 2-factor scheme.
- ARC3D diagonalized Beam-Warming scalar pentadiagonal scheme.
- LU-SGS algorithm.
- D3ADI algorithm with Huang subiteration.
- ARC3D Beam-Warming with Steger-Warming flux split jacobians.
- SSOR algorithm (with subiteration)

Implicit Artificial Boundaries

Implicit Artificial Boundary

$$\begin{bmatrix} A^1 & C^1 \\ C^2 & A^2 \end{bmatrix} \begin{bmatrix} \Delta Q^1 \\ \Delta Q^2 \end{bmatrix} = \begin{bmatrix} R^1\left(Q^1, I_h\left(Q^2\right)\right) \\ R^2\left(Q^2, I_h\left(Q^1\right)\right) \end{bmatrix}$$

$$C^1 = \frac{\partial R^1}{\partial Q^2} \qquad C^2 = \frac{\partial R^2}{\partial Q^1}$$

GMRES: Matrix-Vector Multiplication

Artificial Boundary Linearization

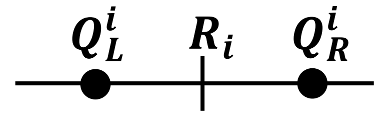
$$C^1 \Delta Q^2 = \frac{\partial R^1\left(Q^1, I_h\left(Q^2\right)\right)}{\partial Q^2} \Delta Q^2$$

Chain Rule

$$C^1 \Delta Q^2 = \frac{\partial R^1\left(Q^1, I_h\left(Q^2\right)\right)}{\partial I_h\left(Q^2\right)} \frac{\partial I_h\left(Q^2\right)}{\partial Q^2} \Delta Q^2$$

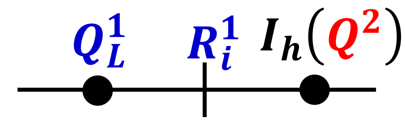
Linear Interpolation Operator

$$\frac{\partial I_h\left(Q^2\right)}{\partial Q^2} \Delta Q^2 = I_h\left(\Delta Q^2\right)$$



Interior Flux Linearization

$$\frac{\partial R_i\left(Q_L^i, Q_R^i\right)}{\partial Q_L} \qquad \frac{\partial R_i\left(Q_L^i, Q_R^i\right)}{\partial Q_R}$$



$$\frac{\partial R^1\left(Q^1, I_h\left(Q^2\right)\right)}{\partial I_h\left(Q^2\right)} = \frac{\partial R_i^1\left(Q_L^1, I_h\left(Q^2\right)\right)}{\partial Q_R^1}$$

Matrix-Vector Product

Receiver Grid Interpolation Mapping

$$C^1 \Delta Q^2 = \frac{\partial R_i^1\left(Q_L^1, I_h\left(Q^2\right)\right)}{\partial Q_R^1} I_h\left(\Delta Q^2\right) = \widetilde{C^1} I_h\left(\Delta Q^2\right)$$

Interior Jacobian Array of Matrices

RHS Interpolation Operator Array of Vectors

Parallel GMRES Iterative Solver

Matrix-Vector Multiplication
Mask Communication with Local Calculations

$$w_{n+1} = Av_n$$

$$\begin{bmatrix} w_{n+1}^1 \\ w_{n+1}^2 \end{bmatrix} = \begin{bmatrix} A^1 & \widetilde{C^1} \\ \widetilde{C^2} & A^2 \end{bmatrix} \begin{bmatrix} v_n^1 \\ v_n^2 \end{bmatrix}$$

Processor 1

Interpolate $I_h(v_n^1)$

Non-Blocking Send $I_h(v_n^1)$

Compute $w_{n+1}^1 = A^1 v_n^1$

Receive $I_h(v_n^2)$

Compute $w_{n+1}^1 = w_{n+1}^1 + \widetilde{C^1} I_h(v_n^2)$

Processor 2

Interpolate $I_h(v_n^2)$

Non-Blocking Send $I_h(v_n^2)$

Compute $w_{n+1}^2 = A^2 v_n^2$

Receive $I_h(v_n^1)$

Compute $w_{n+1}^2 = w_{n+1}^2 + \widetilde{C^2} I_h(v_n^1)$

Compute Local Dot Product ➡ **Dot Products** All Reduce $O\left(\frac{n}{p} + log(p)\right)$

Parallel Efficient If $\frac{n}{p} > log(p)$

Preconditioner Omits C Matrices **Jacobi as p → n**

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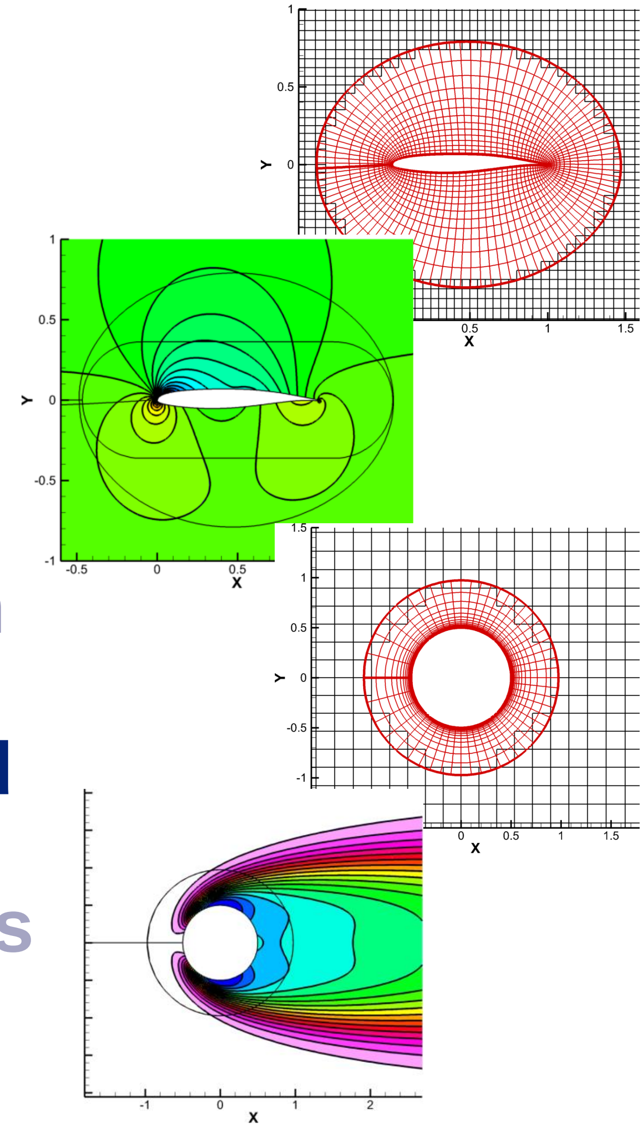

Discontinuous Galerkin Chimera Scheme

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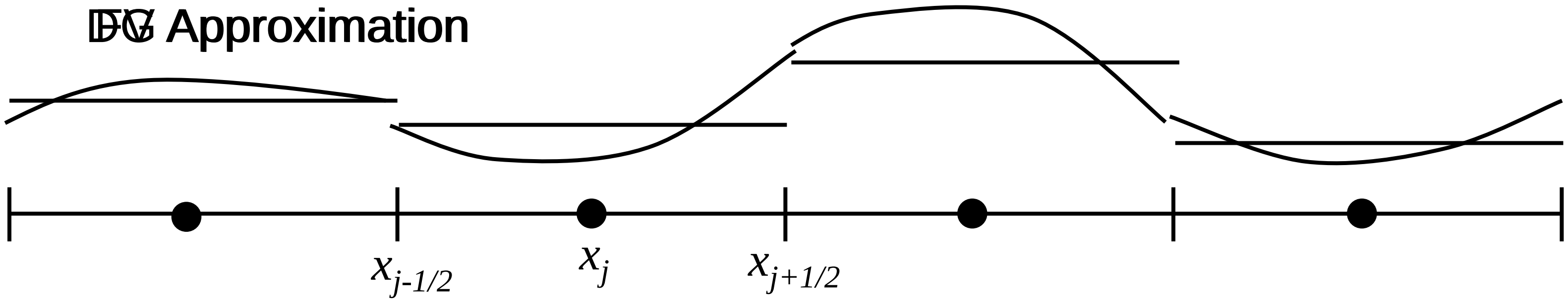
FV DG Approximation



- **Discontinuous Galerkin**
  - **Weak Form**

$$\int_{\Omega_e} \phi \nabla \cdot \vec{F} d\Omega = 0$$

ϕ - Legendre Polynomials

$$R\left(Q^+, Q^-\right) = \int_{\Gamma_e} \phi \vec{F}\left(Q^+, Q^-\right) \cdot \vec{n} d\Gamma - \int_{\Omega_e} \nabla \phi \cdot \vec{F}\left(Q^-\right) d\Omega = 0$$

  - **Approximate Riemann Solver by Roe**
  - **BR2 Viscous Scheme**
- **DG-Chimera**
  - **Natural Interpolation Operator (Solution is Polynomials)**
  - **Curved Elements**
  - **Reduces to a Zonal Interface (Abutting Meshes)**
  - **No Orphan Points due to Fringe Points**

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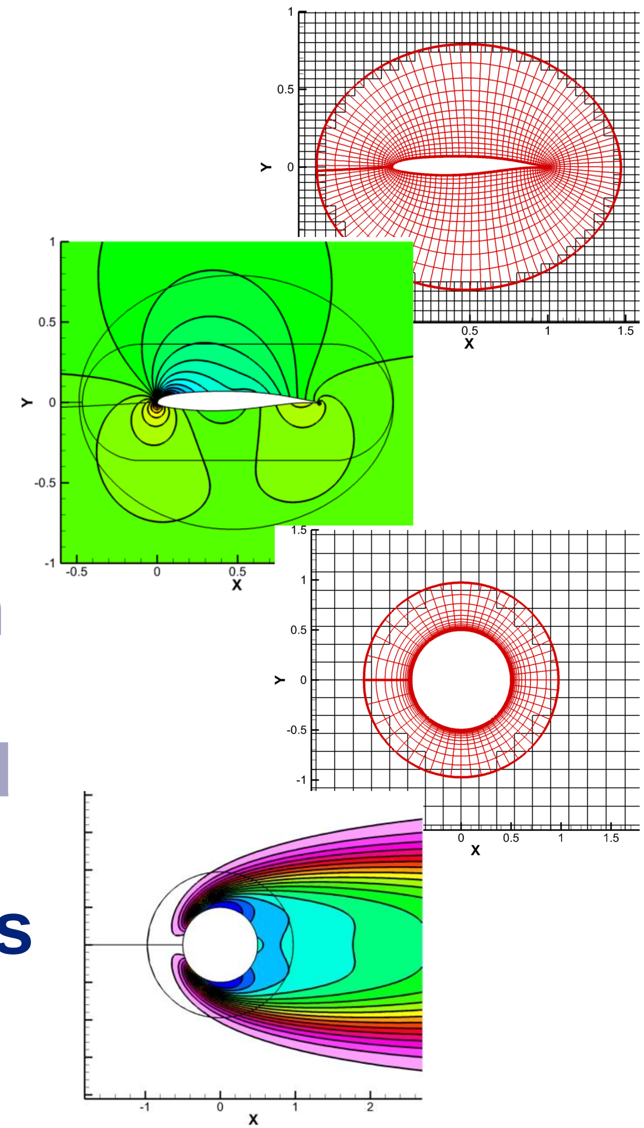
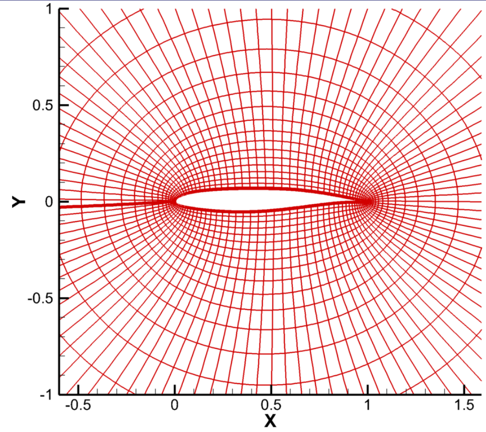
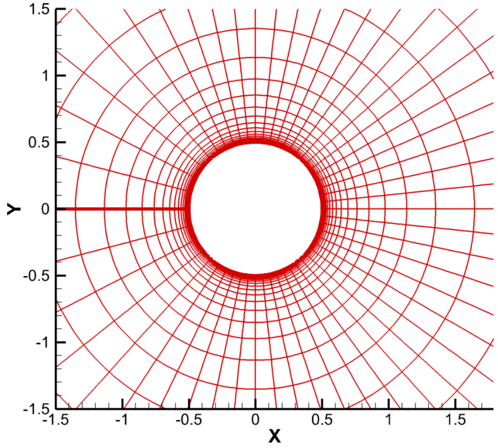


Inviscid/Viscous Flow Examples

- **Inviscid SKF 1.1 Airfoil**
  - $M_\infty = 0.4$
  - $\alpha=2.5°$



- **Viscous Subsonic Circular Cylinder**
  - $M_\infty = 0.25$
  - Re = 40



- **Focus on Solution Time**
  - Explicit vs. Implicit Chimera

Time Integration and Compute Resources

- **Steady State**
  - Quasi-Newton

$$\left(\frac{M}{\Delta t} + \frac{\partial R}{\partial Q}\right)\Delta Q = R$$

$$CFL^{n+1} = CFL^{0}\frac{\|R^{0}\|}{\|R^{n}\|}$$

$$CFL_{max} = 1e30$$

- **GMRES Krylov Solver**
  - ILU1 Preconditioner
  - Converged to 1e-11 Each Newton Iteration

- **Intel Core 2 Duo 3.0 GHz processor 8 GB RAM**
  - 10 Compute Nodes
  - Ethernet Connection

- **MPI Parallelism**
  - Timings for 1, 2, 4, and 8 Processors
  - 1 MPI Process per Node (Maximize Communication)

- **Shared Memory Multi-Threaded**
  - 1 Grid Per Thread

SKF 1.1 Airfoil ($M_\infty = 0.4$, $\alpha=2.5°$) Meshes

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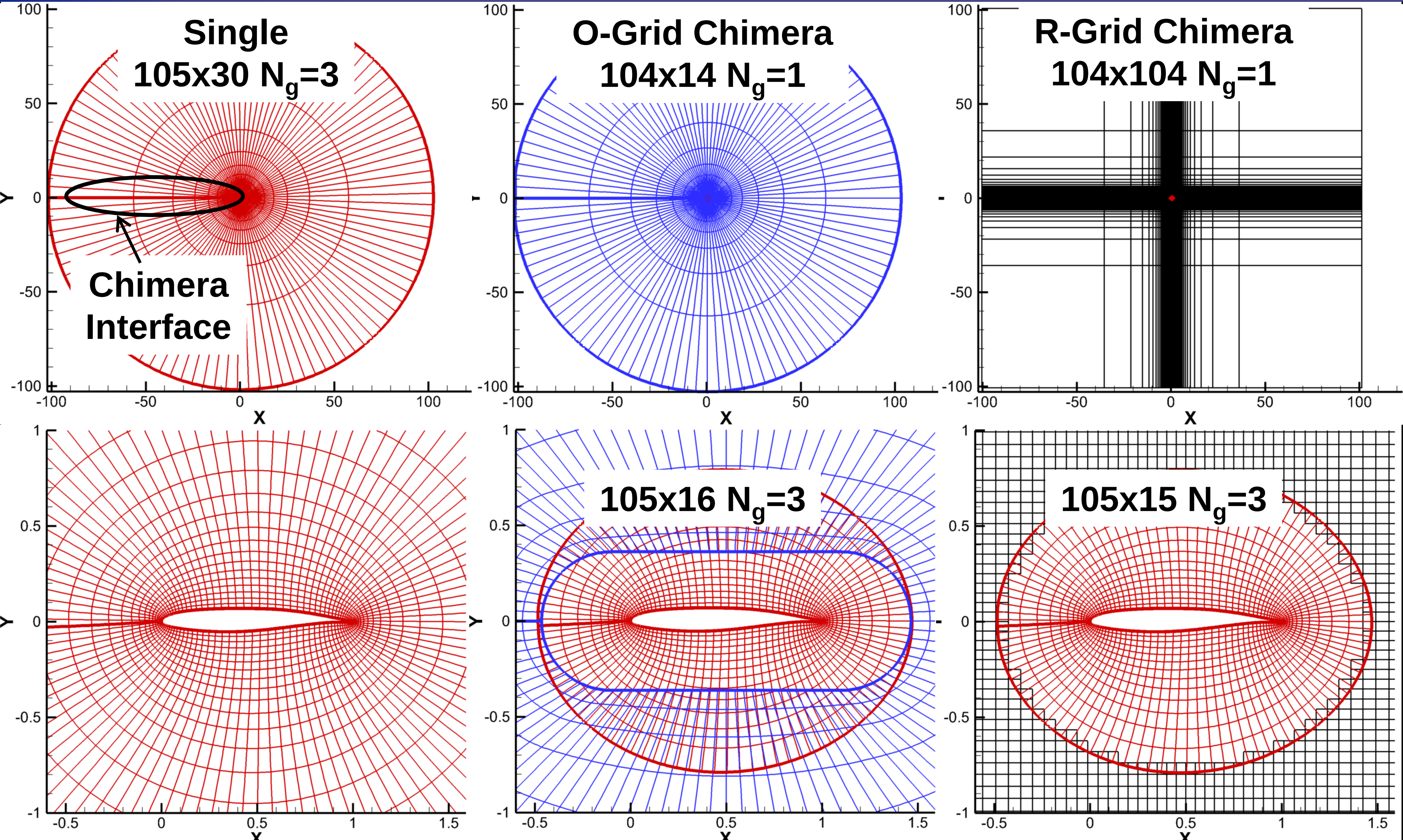

SKF 1.1 Airfoil ($M_\infty = 0.4$, $\alpha=2.5°$) Explicit Chimera Speedup

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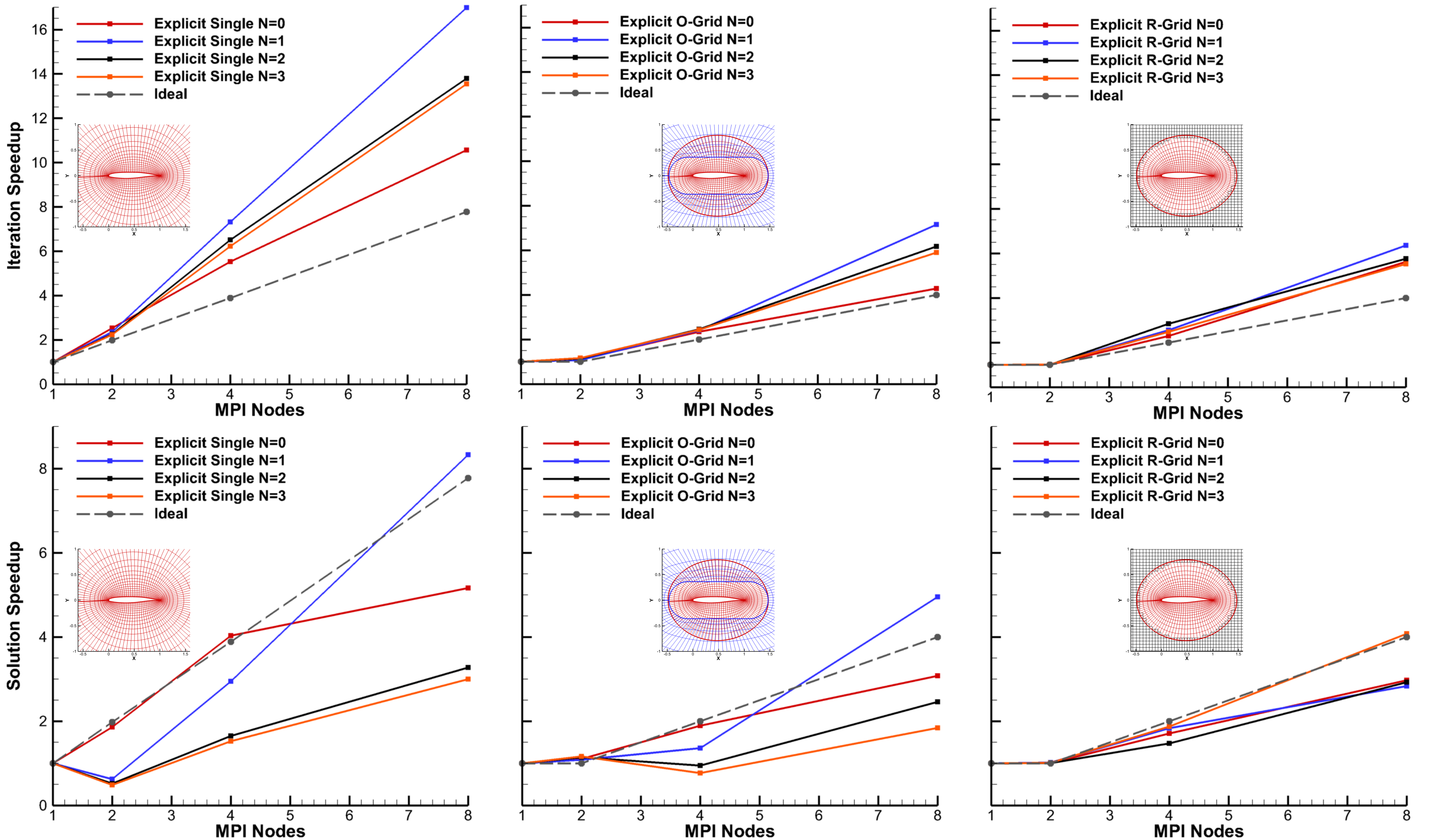
$$A(Q_{Local})\Delta Q = R(Q_{Local}, Q_{Chimera})$$



SKF 1.1 Airfoil ($M_\infty = 0.4$, $\alpha=2.5°$) Explicit Chimera Convergence History

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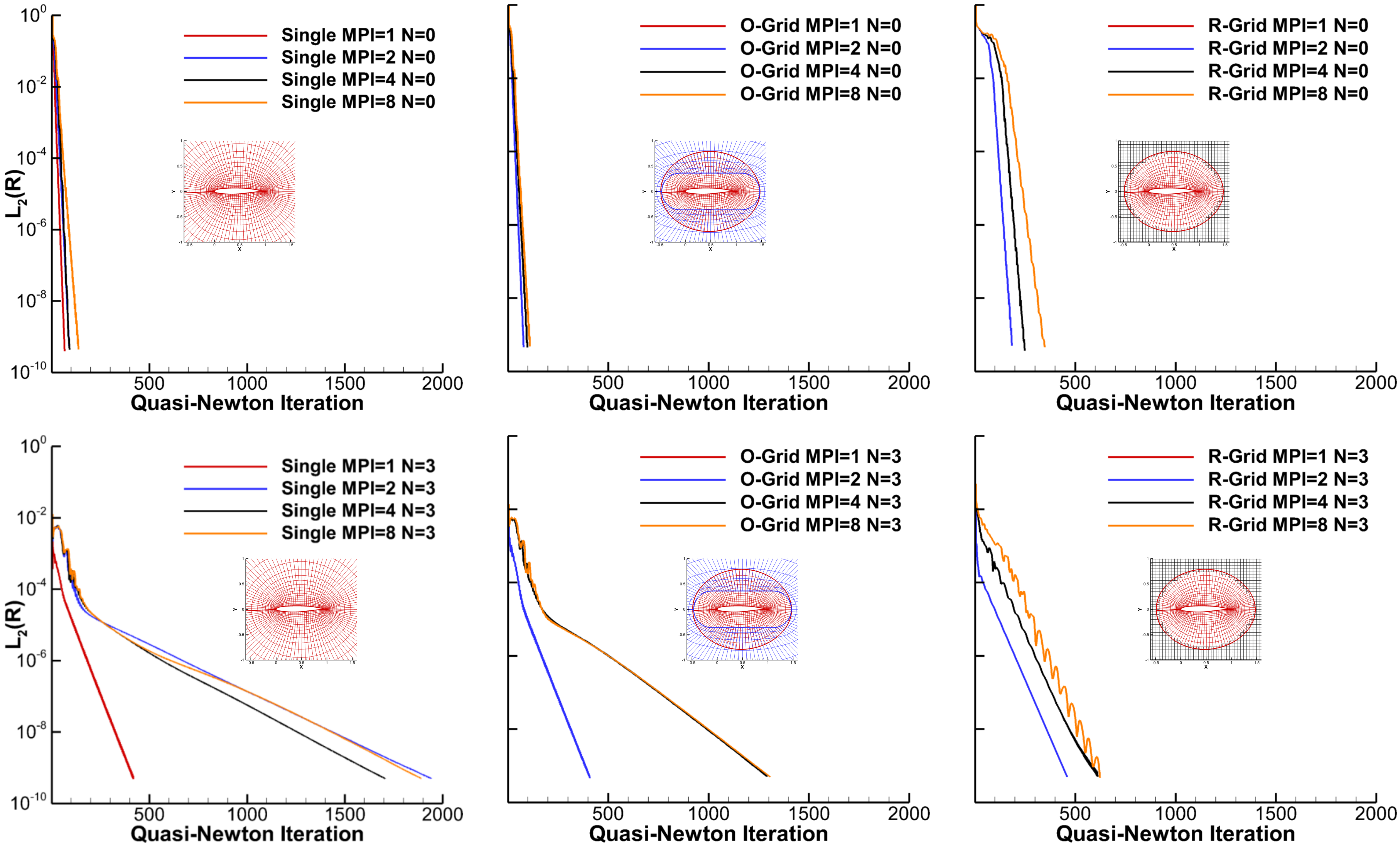
$$A(Q_{Local})\Delta Q = R(Q_{Local}, Q_{Chimera})$$



SKF 1.1 Airfoil ($M_\infty = 0.4$, $\alpha=2.5°$) Implicit Chimera Speedup

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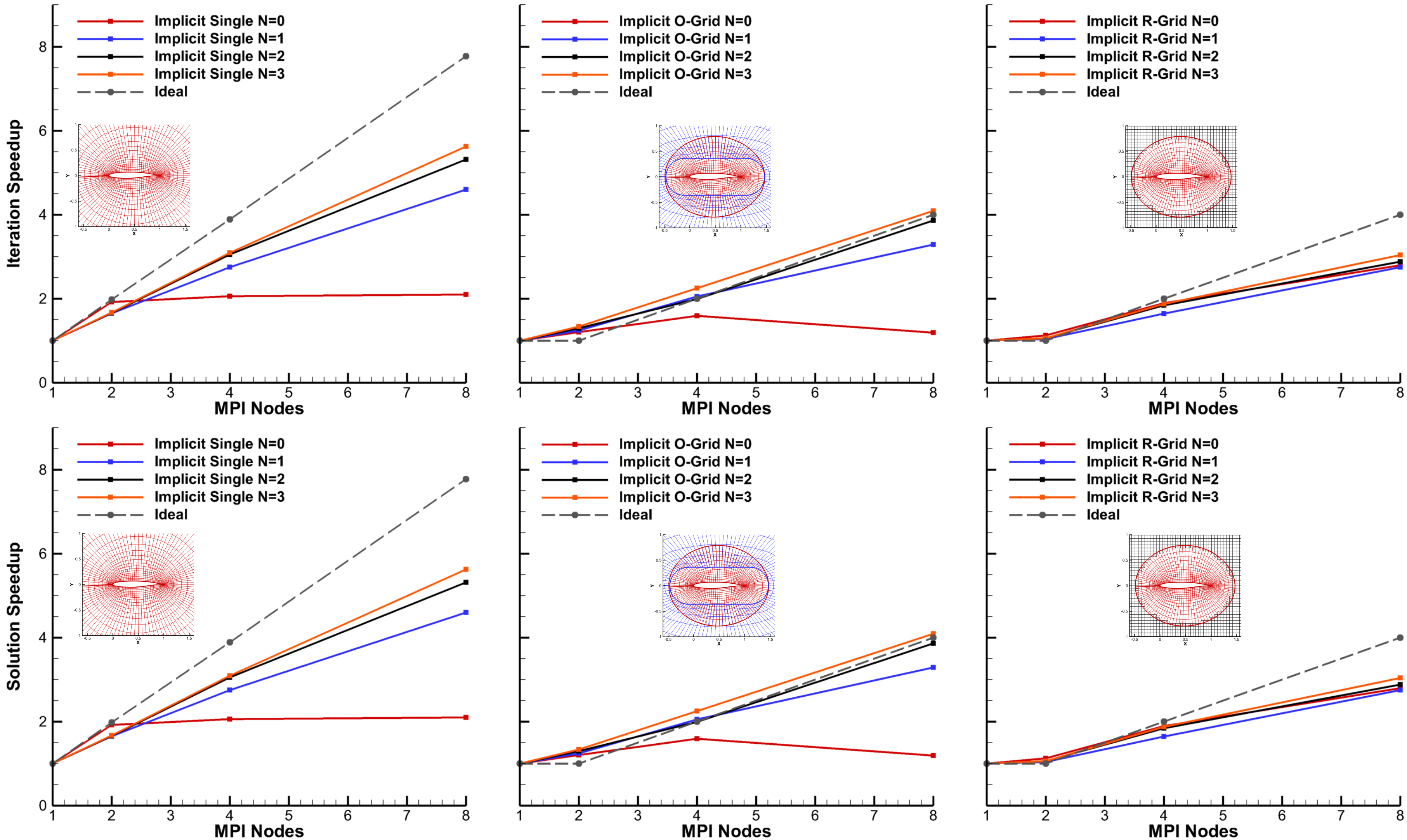
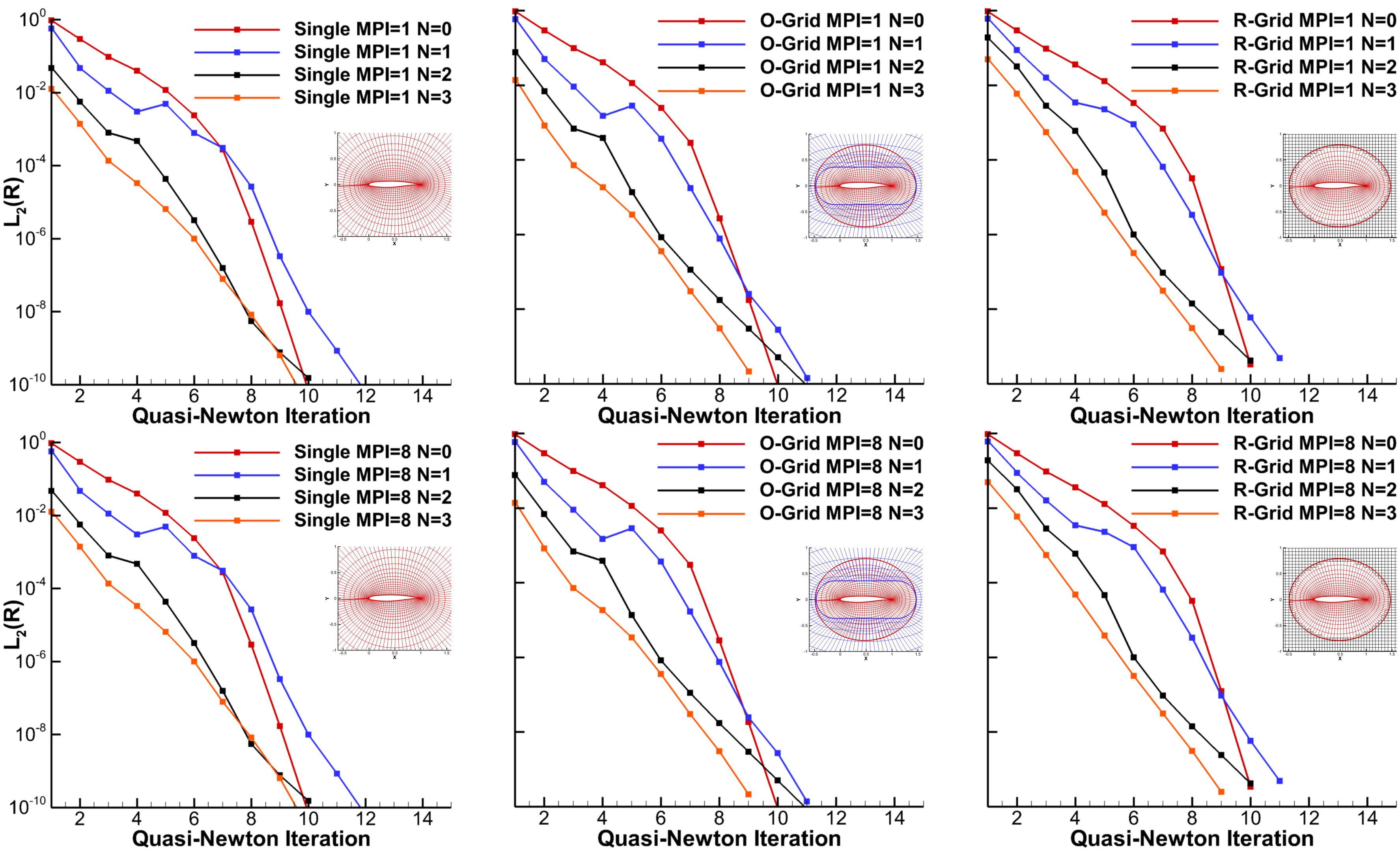
$$A(Q_{Local}, Q_{Chimera})\Delta Q = R(Q_{Local}, Q_{Chimera})$$



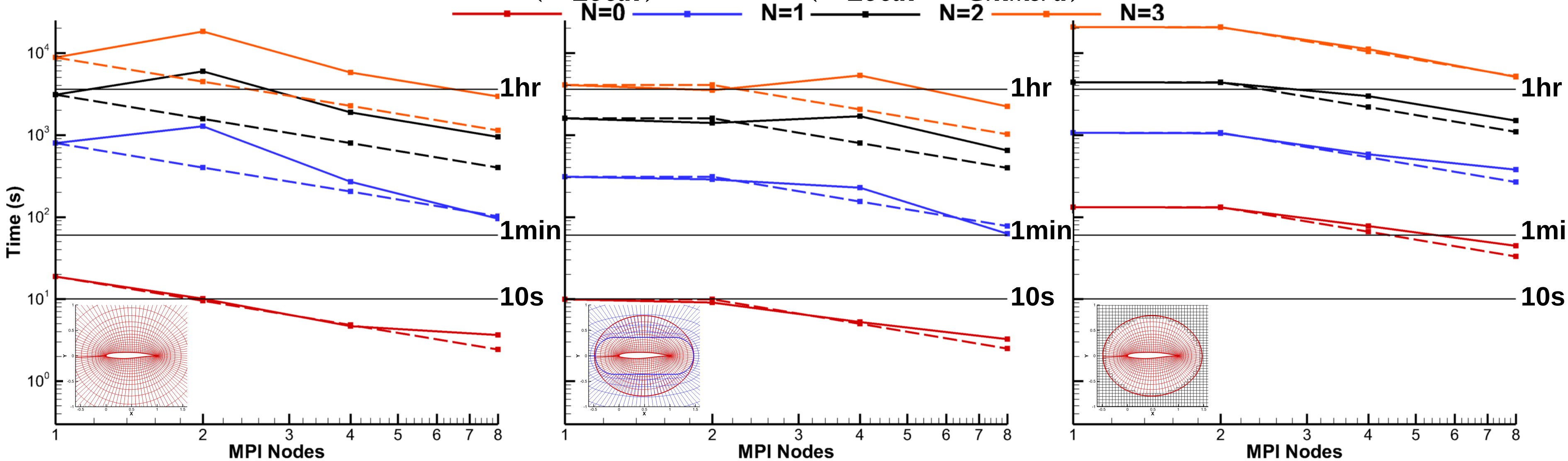
SKF 1.1 Airfoil ($M_\infty = 0.4$, $\alpha=2.5°$)
Implicit Convergence History

$$A\left(Q_{Local}, Q_{Chimera}\right)\Delta Q = R\left(Q_{Local}, Q_{Chimera}\right)$$

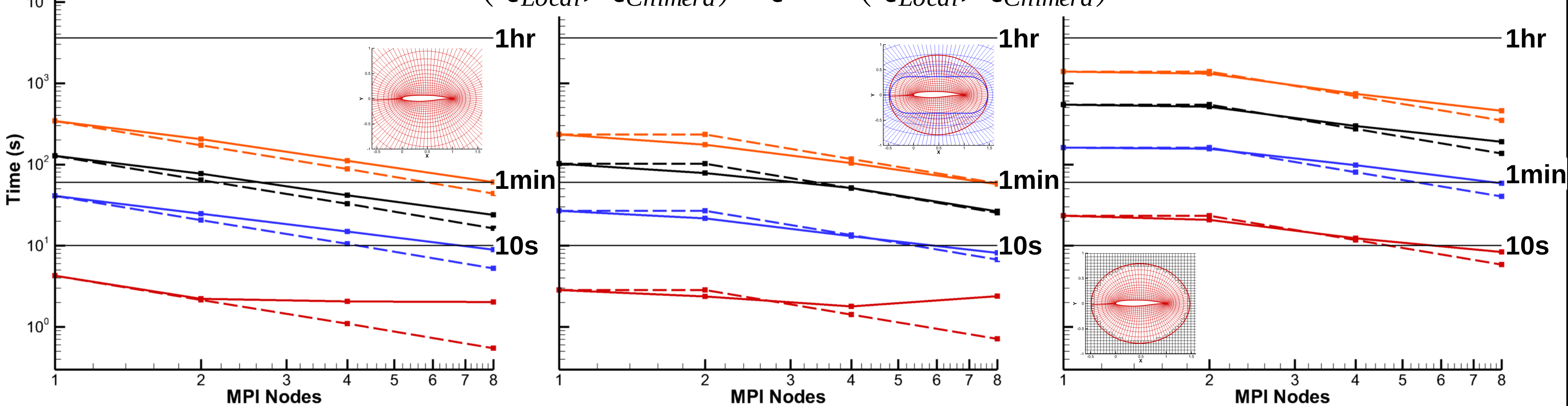


SKF 1.1 Airfoil ($M_\infty = 0.4$, α=2.5°) Solution Time

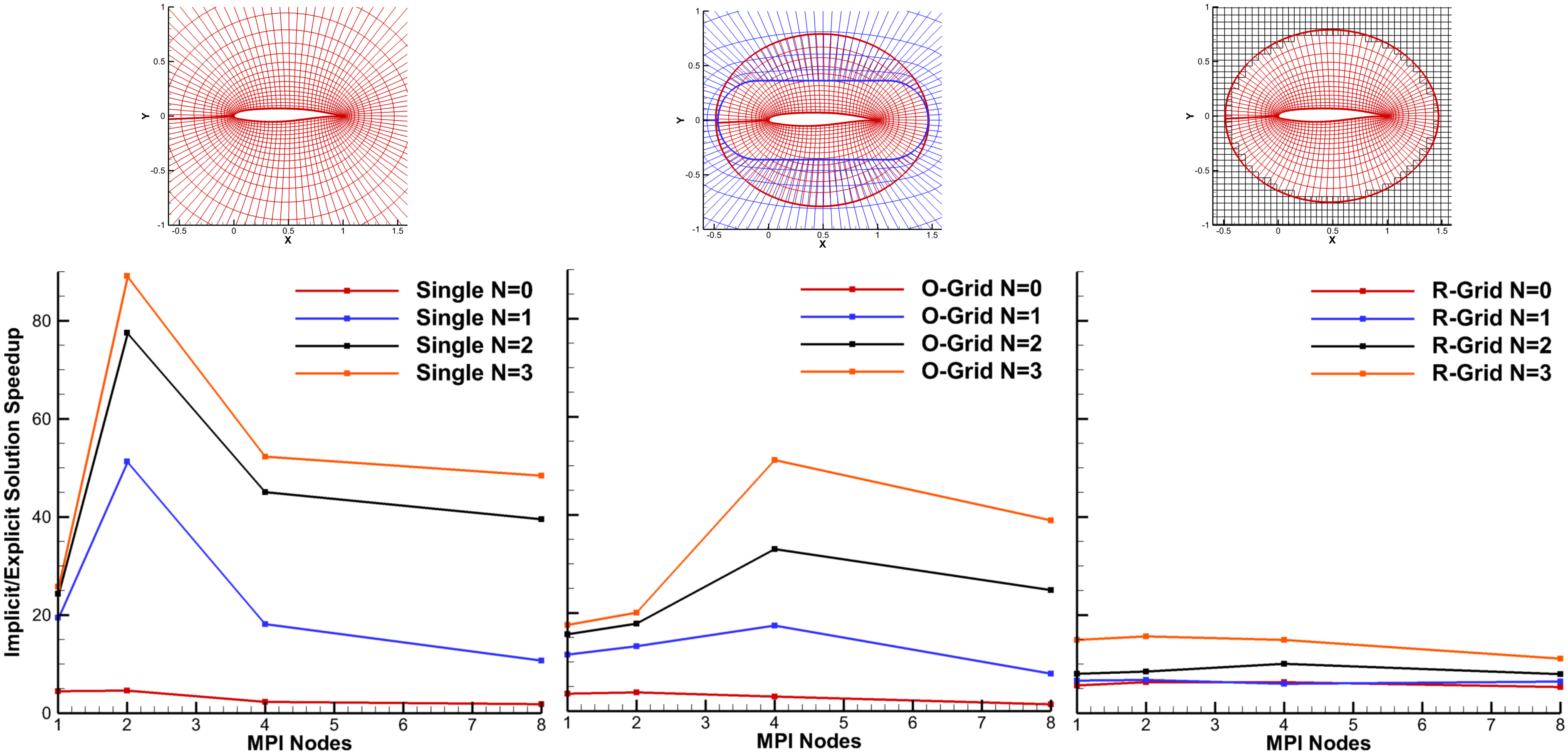
$$A\left(Q_{Local}\right)\Delta Q = R\left(Q_{Local}, Q_{Chimera}\right)$$



$$A\left(Q_{Local}, Q_{Chimera}\right)\Delta Q = R\left(Q_{Local}, Q_{Chimera}\right)$$



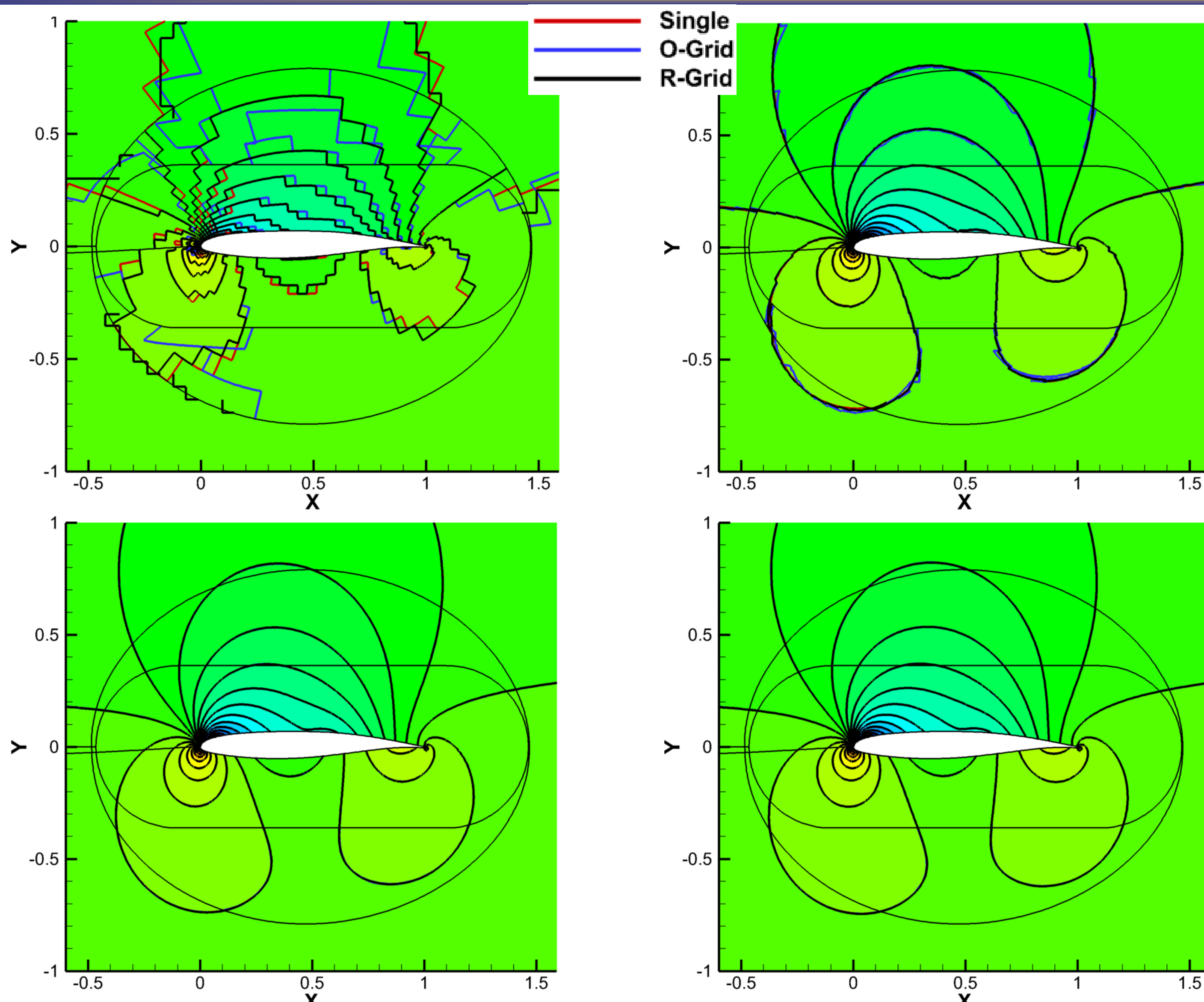
SKF 1.1 Airfoil ($M_\infty = 0.4$, $\alpha=2.5°$)
Implicit/Explicit Solution Speedup



SKF 1.1 Airfoil ($M_{\infty} = 0.4$, $\alpha=2.5°$)
Cp Contour Lines

N=0
1st-order

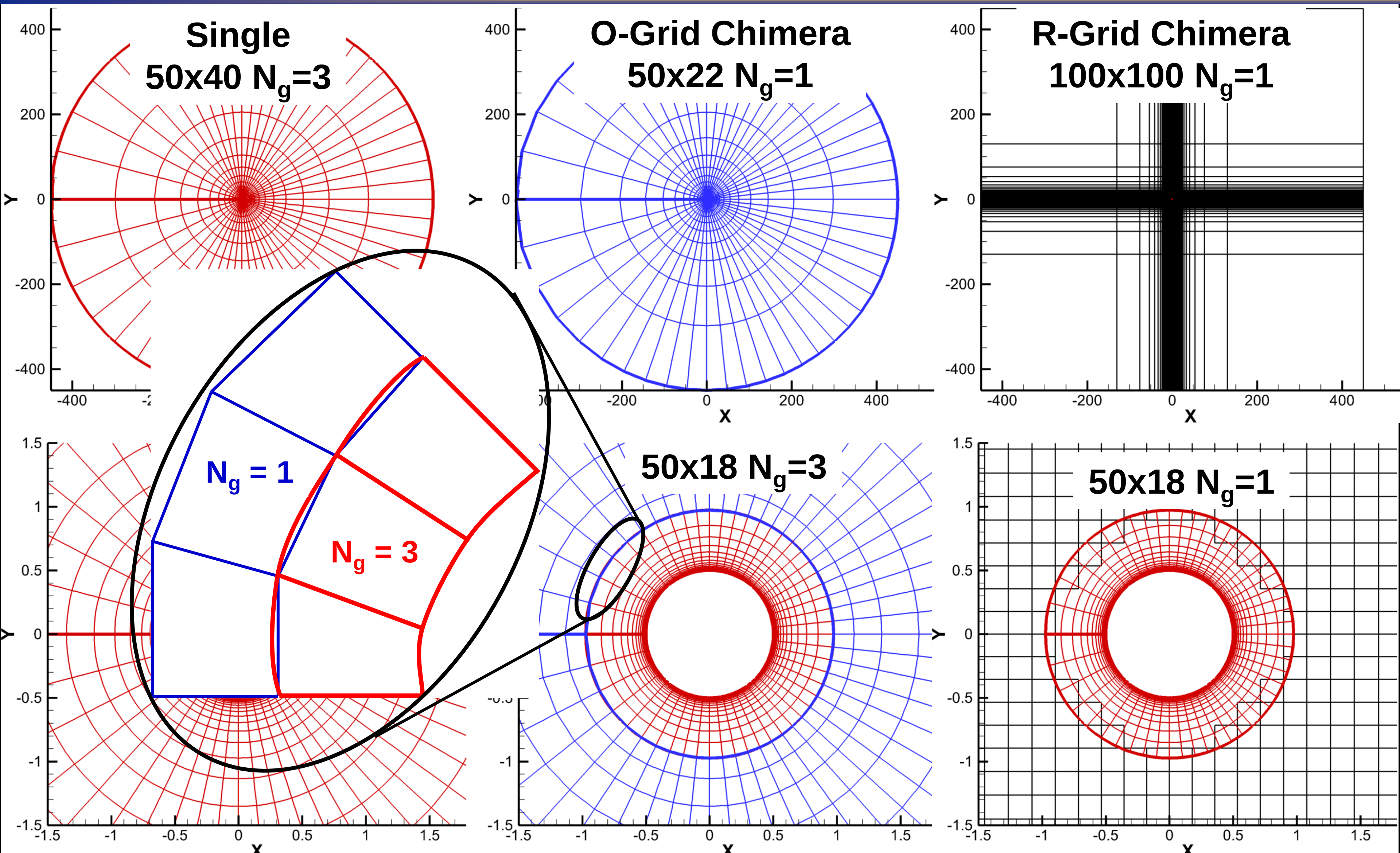
N=1
2nd-order



N=2
3rd-order

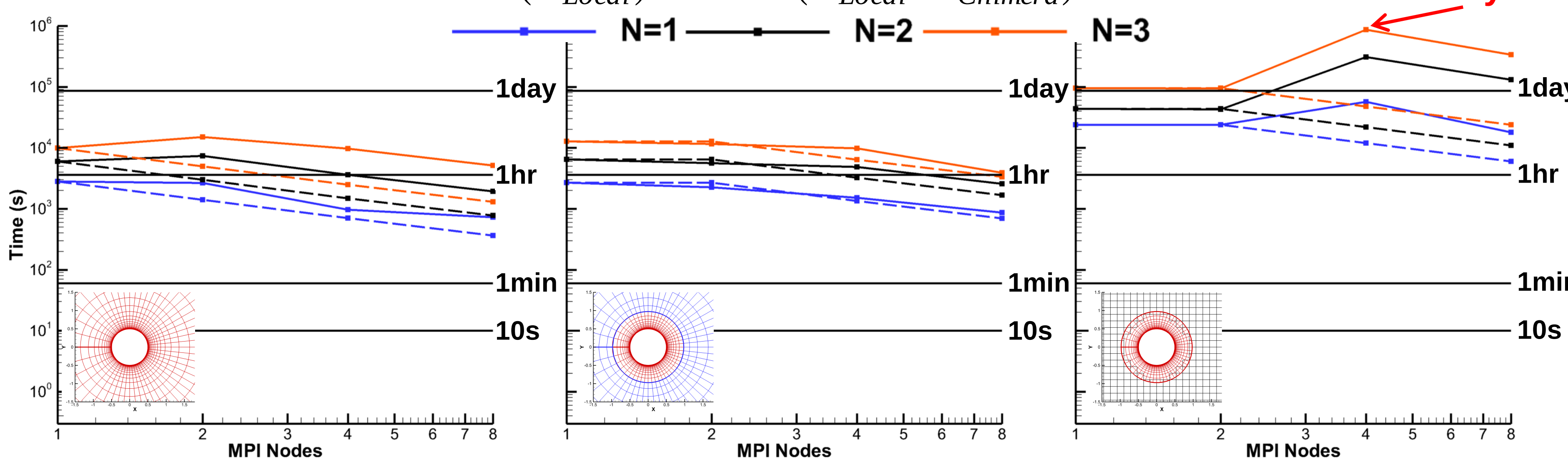
N=3
4th-order

Circular Cylinder (Re = 40) Meshes

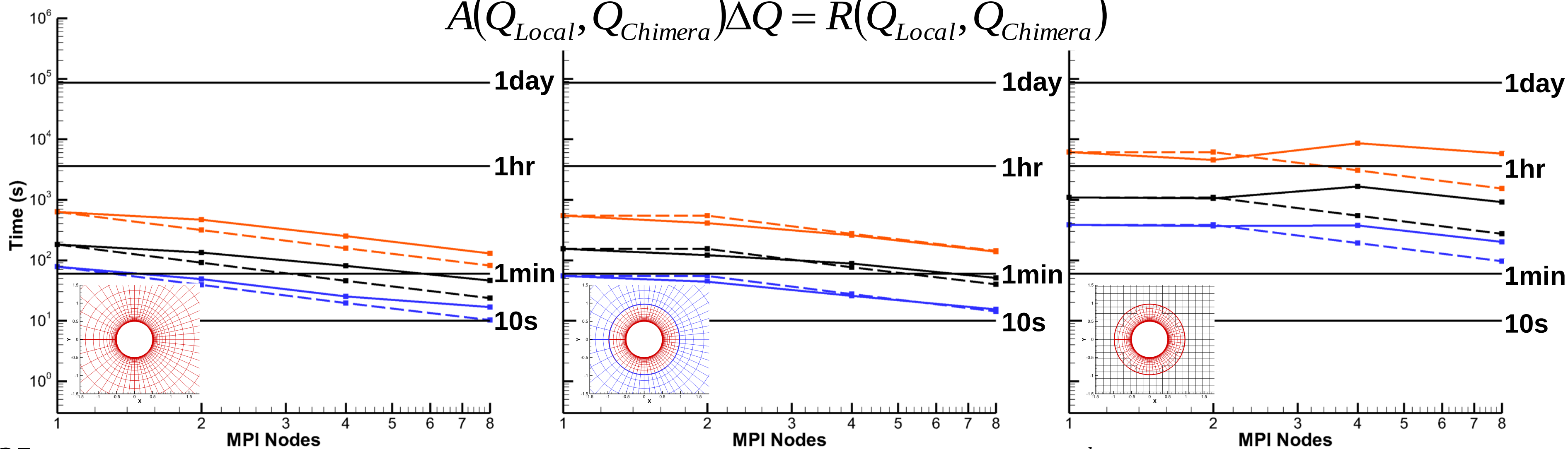


Circular Cylinder (Re = 40) Solution Time

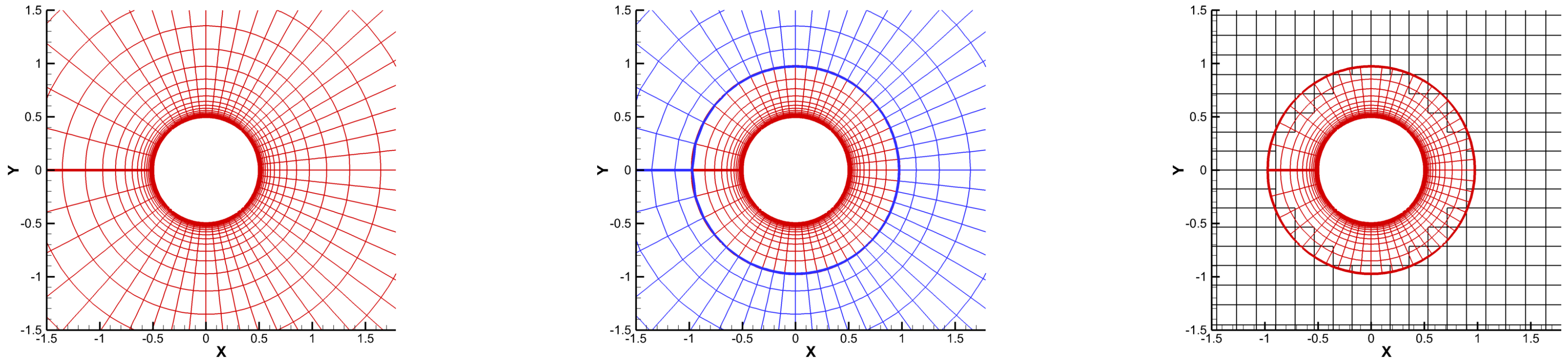
$$A\left(Q_{Local}\right)\Delta Q = R\left(Q_{Local}, Q_{Chimera}\right)$$

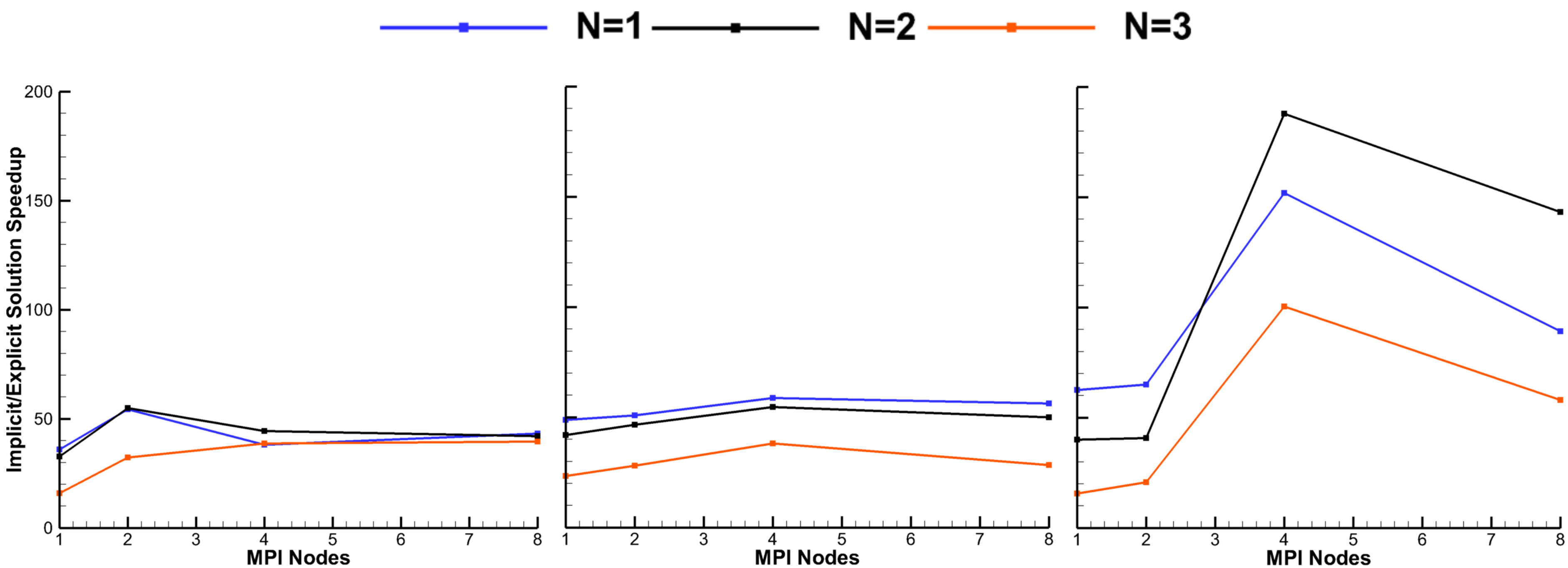


$$A\left(Q_{Local}, Q_{Chimera}\right)\Delta Q = R\left(Q_{Local}, Q_{Chimera}\right)$$

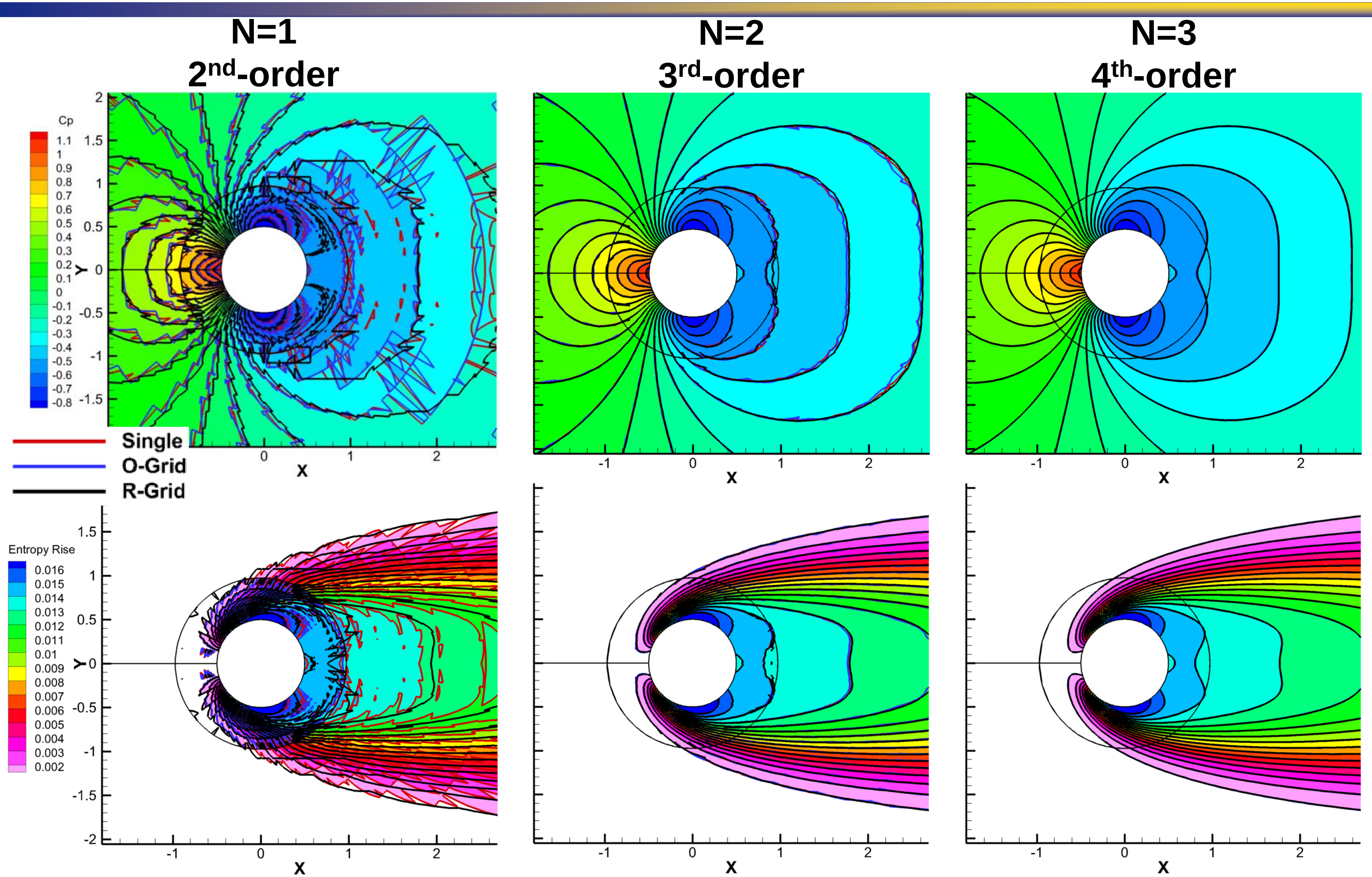


Circular Cylinder (Re = 40)
Implicit/Explicit Solution Speedup





Subsonic Circular Cylinder ($M_\infty = 0.25$)
Cp/Entropy Rise Contour Lines



Conclusion and Future Work

- **Implicit Artificial Boundaries**
  - Included with GMRES Matrix-Vector Multiplication
  - Omitted in Preconditioner
  - Minimal Information Communicated
  - Significantly Reduces Execution Time

- **Few Modifications Required to Existing Codes**
  - ~95% of Code already Exists
  - Spares Matrix-Vector Multiplication
  - Restarted GMRES Fortran Code
    - http://people.sc.fsu.edu/~jburkardt/f_src/mgmres/mgmres.html

- **Demonstrated on Inviscid/Viscous Flows**

Thank you!
Questions?

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