Solving Fluid Structure Interaction Problems with Overture

Overcoming the fluid-structure added-mass instability for incompressible flows

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12th Symposium on Overset Composite Grids and Solution Technology,
Georgia Institute of Technology, Atlanta, Georgia, October 6–9, 2014.

Acknowledgments.

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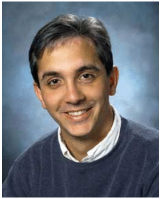

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Supported by:

Department of Energy, Office of Science, ASCR Applied Math Program.
LLNL: Laboratory Directed Research and Development (LDRD) program.
National Science Foundation, RPI.

Overture and the CG solvers are open source

The Overture framework and Composite Grid (CG) PDE solvers are open source and available from (documentation, downloads)

overtureFramework.org

The source-code repositories are hosted at

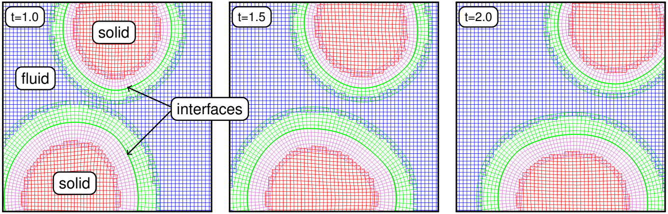
sourceforge.net/projects/overtureframework

Fluid Structure Interaction (FSI) algorithms

Our approach in Overture is based on deforming composite grids.

Approach:

- Fluids are solved in an Eulerian frame.
- Solids are solved in a Lagrangian frame.
- Deforming interface grids are regenerated at each time-step with the hyperbolic grid generator.



Basic Components used in FSI Time-Stepping.

1. HyperbolicMapping - hyperbolic component grid generator (to recompute grids next to deforming surfaces).
2. Ogen grid generator (to re-compute connectivity).
3. Fluid solver (e.g. Cgcns, Cgins).
4. Solid Solver (e.g. Cgsm, BeamSolver, . . .)
5. Multi-physics, multi-domain driver program (Cgmp)

Basic FSI Algorithm.

procedure SOLVEFSIDCG($\mathcal{G}$, t_{final}) ▷ Input: initial composite grid and final time
    $t := 0$; $n := 0$; $\mathcal{G}^n = \mathcal{G}$;
    assignInitialConditions($\mathbf{q}_\mathbf{i}^n$, $\bar{\mathbf{q}}_\mathbf{i}^n$, $\mathcal{G}^n$);
    while $t < t_{\text{final}}$ **do**
        $\Delta t :=$ computeTimeStep($\mathbf{q}_\mathbf{i}^n$, $\bar{\mathbf{q}}_\mathbf{i}^n$, $\mathcal{G}^n$);
        $\mathcal{G}^p :=$ moveGrids($\mathcal{G}^n$, $\mathbf{q}_\mathbf{i}^n$, $\bar{\mathbf{q}}_\mathbf{i}^n$); ▷ (calls HyperbolicMapping)
        $\mathcal{G}^p :=$ updateOverlappingGrid($\mathcal{G}^p$); ▷ (Ogen)
        $\mathbf{q}_\mathbf{i}^{n+1} :=$ advanceFluid($\mathbf{q}_\mathbf{i}^n$, $\mathcal{G}^n$, $\mathcal{G}^p$, Δt);
        $\bar{\mathbf{q}}_\mathbf{i}^{n+1} :=$ advanceSolid($\bar{\mathbf{q}}_\mathbf{i}^n$, $\mathcal{G}$, Δt);
        $(\mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I) :=$ projectInterface($\mathbf{q}_\mathbf{i}^{n+1}$, $\bar{\mathbf{q}}_\mathbf{i}^{n+1}$, $\mathcal{G}^p$);
        $\mathbf{q}_\mathbf{i}^{n+1} :=$ applyFluidBCs($\mathbf{q}_\mathbf{i}^{n+1}$, $\mathcal{G}^p$, $\mathbf{n}^T\mathbf{v}^I$, $\mathbf{n}^T\boldsymbol{\sigma}^I$);
        $\bar{\mathbf{q}}_\mathbf{i}^{n+1} :=$ applySolidBCs($\bar{\mathbf{q}}_\mathbf{i}^{n+1}$, $\mathcal{G}$, $\mathbf{n}^T\mathbf{v}^I$, $\mathbf{n}^T\boldsymbol{\sigma}^I$);
        $\mathcal{G}^{n+1} :=$ correctMovingGrids($\mathbf{q}_\mathbf{i}^{n+1}$, $\bar{\mathbf{q}}_\mathbf{i}^{n+1}$, $\mathcal{G}^p$, Δt);
        $t := t + \Delta t$; $n := n + 1$;
    end while
end procedure

Movies: Incompressible flow with rigid bodies

Traditional algorithms fail for light solids

Incompressible flow + rigid cyls

drops

drops-speed.mp4

Incompressible flow + stick

drop stick

dropStick.mp4

FSI and Added-Mass Instabilities

Traditional partitioned algorithms may require many sub-time-step iterations for light solids.

Monolithic schemes are stable but can be expensive, less flexible.

Traditional partitioned algorithm:

1. Advance solid using stress (traction) from the fluid

$$\sigma^I = \sigma_{fluid}$$

2. Advance fluid using velocity from the solid,

$$v^I = \bar{v}_{solid}$$

The stability problem is particularly acute for incompresssible flows (e.g. blood flow).

The added-mass instability has received much attention in the literature

Approaches to partially address the added mass instability:

- Robin-Robin (mixed) boundary conditions with coefficients determined from simplified known solutions.
- interface artificial compressibility, fictitious pressure and fictitious mass.
- fixed point iterations (Aitken accelerated).
- semi-monolitic, approximate factorizations and Newton type schemes.

Causin, Gerbeau, Nobile (2005), Forster, Wall, Ramm (2007), vanBrummelen (2009) Badia, Quaini, Quarteroni (2008), Astorino, Chouly, Fernandez (2009), Degroote, Bathe, Vierendeels (2009), Guidoboni, Glowinski, Cavallini, Canic (2009) Fernandez (review, 2011), Gretarsson, Kwatra, Fedkiw (2011) Baek Karniadakis (2012) Nobile, Vergara (2012), Yu, Baek, Karniadakis (2013), Bukac, Canic, Glowinski, Tambaca, Quaini (2013), Fernandez, Mullaert, Vidrascu (2014) Fernandez, Landajuela (2014), . . .

Added Mass Partitioned (AMP) Algorithms

Overcome the added-mass instability - require no sub-iterations

In some recent papers we have shown how to over-come the added-mass instability for different FSI regimes (AMP schemes require no sub-iterations)

1. Compressible flow + compressible solids
   $\rightarrow$ Embed solution of a fluid-solid Riemann problem.
2. Compressible flow + rigid solids
   $\rightarrow$ Incorporate added-mass tensors into the rigid body equations.
3. Incompressible flow + beams/shells
   $\rightarrow$ Derive Robin pressure boundary condition by matching accelerations.
4. Incompressible flow + compresssible elastic bulk solids
   $\rightarrow$ Derive Robin pressure boundary condition from solid characteristics.

- JWB, B. Sjögreen, *A normal mode stability analysis of numerical interface conditions for fluid/structure interaction*, Commun. Comput. Phys. (2011).
- JWB, WDH, DWS, *Deforming Composite Grids for Solving Fluid Structure Problems*, JCP (2012).
- JWB, WDH, BS, *A stable FSI algorithm for light rigid bodies in compressible flow*, JCP (2013).
- JWB, WDH, DWS, *An analysis of a new stable partitioned algorithm for FSI problems. Part I: Incompressible flow and elastic solids*, JCP (2014)
- JWB, WDH, DWS, *An analysis of a new stable partitioned algorithm for FSI problems. Part II: Incompressible flow and structural shells*, JCP (2014).

AMP: Compressible flow + elastic solids

The AMP scheme for compressible flow and elastic solids is based on a impedance weight average of provisional fluid and solid values:

$$
\begin{aligned}
v_I &= \frac{\bar{z}\bar{v} + zv}{\bar{z} + z} + \frac{\sigma - \bar{\sigma}}{\bar{z} + z}, \\
\sigma_I &= \frac{\bar{z}^{-1}\bar{\sigma} + z^{-1}\sigma}{\bar{z}^{-1} + z^{-1}} + \frac{v - \bar{v}}{\bar{z}^{-1} + z^{-1}}
\end{aligned}
$$

where $\bar{z} = \bar{\rho}\bar{c}_p$ and $z = \rho a$ are the solid and fluid impedances. Derived from a fluid-solid Riemann problem.

Compressible flow + elastic solids

Shock hitting a neo-Hookean solid

Neo-Hookean solid

deformingEllipseNeoHookean.mp4

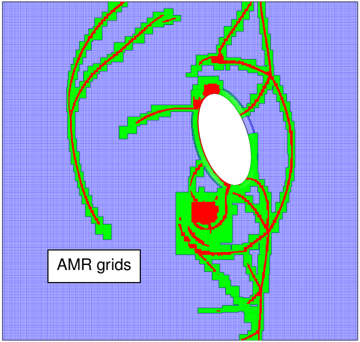
Shock hitting two elastic sticks

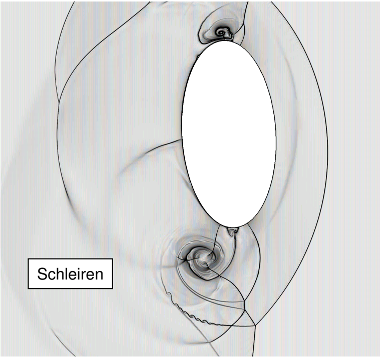
deforming sticks

deformingSticks.mp4

AMP: Compressible flow + light rigid-bodies

Approach: Analytically derive *exact* added-mass tensors for the rigid-body equations of motion.





Shock hitting an ellipse of *zero* mass.

- J.W. Banks, WDH, Sjögreen, *A stable FSI algorithm for light rigid bodies in compressible flow*, JCP (2013).

Compressible flow + elastic solids

Shock hitting a zero mass ellipse

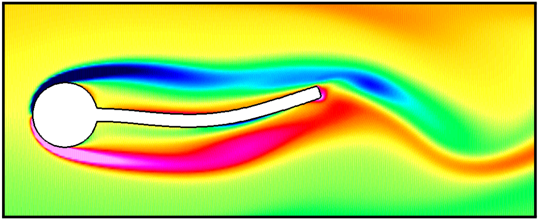
zero mass ellipse

shockMassZeroEllipse.mp4

FSI for Incompressible Flows

Traditional partitioned schemes have historically suffered from added mass instabilities for light solids

Incompressible flows are particularly difficult due to the infinite speed of sound.



Flapping beam (requires tens of sub-iterations per time step with traditional scheme).

1. Traditional FSI partitioned schemes often fail for light solids and require multiple sub-iterations per time-step.
2. The problem originates with the *added mass* effect – to move a body one must also move the surrounding fluid.
3. Incompressible flows are important (e.g. blood flow in a vein, flapping flag, underwater structures) but are particularly difficult.

Incompressible flow with Beams

Using traditional scheme and many (20-40) sub time-step iterations.

Incompressible flow + flexible beam

two beams

`twoBeamsInAChannelCrop.mp4`

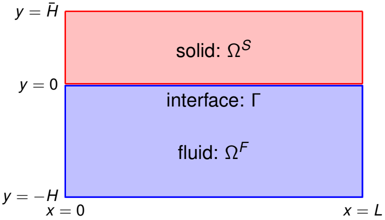
cyl beam

`cylBeamTH3f4Crop.mp4`

Incompressible flow + two beams

Incompressible Stokes fluid + elastic bulk solid.

For analysis and evaluation we consider an FSI model problem.



Incompressible Stokes fluid and compressible elastic bulk solid equations:

$$\text{Fluid:} \begin{cases} \rho \dfrac{\partial \mathbf{v}}{\partial t} + \nabla p = \mu \Delta \mathbf{v}, & \mathbf{x} \in \Omega^F, \\ \nabla \cdot \mathbf{v} = 0, & \mathbf{x} \in \Omega^F, \\ \mathbf{v}(x, -H, t) = 0, \end{cases} \qquad \text{Solid:} \begin{cases} \bar{\rho} \dfrac{\partial^2 \bar{\mathbf{u}}}{\partial t^2} = (\bar{\lambda} + \bar{\mu}) \nabla(\nabla \cdot \bar{\mathbf{u}}) + \bar{\mu} \Delta \bar{\mathbf{u}}, & \mathbf{x} \in \Omega^S, \\ \bar{\mathbf{u}}(x, \bar{H}, t) = 0, \end{cases}$$

$$\text{Interface:} \qquad \mathbf{v} = \frac{\partial \bar{\mathbf{u}}}{\partial t},$$

$$\mu\left(\frac{\partial v_1}{\partial y} + \frac{\partial v_2}{\partial x}\right) = \bar{\mu}\left(\frac{\partial \bar{u}_1}{\partial y} + \frac{\partial \bar{u}_2}{\partial x}\right), \quad -p + 2\mu \frac{\partial v_2}{\partial y} = \bar{\lambda} \nabla \cdot \bar{\mathbf{u}} + 2\bar{\mu} \frac{\partial \bar{u}_2}{\partial y}, \quad \mathbf{x} \in \Gamma.$$

Robin condition is derived from outgoing solid characteristic.

Key ingredient I: Given predicted values for the solid, use the out-going solid characteristic variables, to define the fluid interface conditions

$$\mathbf{n}^T\boldsymbol{\sigma}\mathbf{n} + \bar{z}_p\mathbf{n}^T\mathbf{v} = \mathbf{n}^T\bar{\boldsymbol{\sigma}}^{(p)}\mathbf{n} + \bar{z}_p\mathbf{n}^T\bar{\mathbf{v}}^{(p)} \equiv \mathcal{B}(\bar{\boldsymbol{\sigma}}^{(p)}, \bar{\mathbf{v}}^{(p)}), \qquad \mathbf{x} \in \Gamma$$

$$\mathbf{e}_m^T\boldsymbol{\sigma}\mathbf{n} + \bar{z}\mathbf{e}_m^T\mathbf{v} = \mathbf{e}_m^T\bar{\boldsymbol{\sigma}}^{(p)}\mathbf{n} + \bar{z}\mathbf{e}_m^T\bar{\mathbf{v}}^{(p)} \equiv \mathcal{B}_m(\bar{\boldsymbol{\sigma}}^{(p)}, \bar{\mathbf{v}}^{(p)}), \qquad \mathbf{x} \in \Gamma$$

$\bar{z}_p = \bar{\rho}\bar{c}_p$ and $\bar{z} = \bar{\rho}\bar{c}_s$ are the solid impedances , OR since $\boldsymbol{\sigma} = -pI + \boldsymbol{\tau}$,

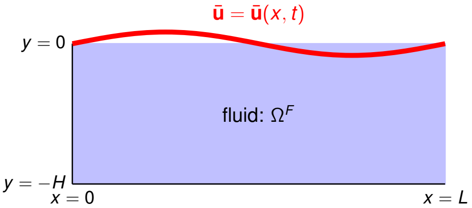
$$-p + \mathbf{n}^T\boldsymbol{\tau}\mathbf{n} + \bar{z}_p\mathbf{n}^T\mathbf{v} = \mathcal{B}(\bar{\boldsymbol{\sigma}}^{(p)}, \bar{\mathbf{v}}^{(p)}), \qquad \mathbf{x} \in \Gamma,$$

$$\mathbf{e}_m^T\boldsymbol{\tau}\mathbf{n} + \bar{z}\mathbf{e}_m^T\mathbf{v} = \mathcal{B}_m(\bar{\boldsymbol{\sigma}}^{(p)}, \bar{\mathbf{v}}^{(p)}), \qquad m = 1, 2, \qquad \mathbf{x} \in \Gamma,$$

Key ingredient II: Matching *accelerations* instead of *velocities* gives a Robin condition for the fluid pressure

$$\boxed{-p - \frac{\bar{z}_p\Delta t}{\rho}\frac{\partial p}{\partial n}} + \mathbf{n}^T\boldsymbol{\tau}\mathbf{n} + \frac{\mu\bar{z}_p\Delta t}{\rho}\mathbf{n}^T(\Delta\mathbf{v}) = \mathbf{n}^T\bar{\boldsymbol{\sigma}}^{(p)}\mathbf{n} + \bar{z}_p\Delta t\,\mathbf{n}^T\frac{\partial\bar{\mathbf{v}}^{(p)}}{\partial t}, \qquad \mathbf{x} \in \Gamma$$

Incompressible Stokes fluid + beam/shell.



Incompressible Stokes fluid and beam equations:

Fluid: $$\begin{cases} \rho \dfrac{\partial \mathbf{v}}{\partial t} + \nabla p = \mu \Delta \mathbf{v}, & \mathbf{x} \in \Omega^F, \\ \nabla \cdot \mathbf{v} = 0, & \mathbf{x} \in \Omega^F, \\ \mathbf{v}(x,-H,t) = 0, \end{cases}$$

Solid: $$\left\{ \bar{\rho} h_s \bar{\mathbf{v}}_t = \bar{\mathbf{L}}(\bar{\mathbf{u}}) - \boldsymbol{\sigma}\mathbf{n}, \quad \mathbf{x} \in \Gamma, \right.$$

Interface: $$\mathbf{v} = \frac{\partial \bar{\mathbf{u}}}{\partial t}$$

Example beam operator: $\bar{\mathbf{L}}(\bar{\mathbf{u}}) = -EI\, \bar{\mathbf{u}}_{xxxx}$.

AMP schemes for INS + beams or shells

Robin condition derives from matching *accelerations*

Beam equation for solid

$$\bar{\rho}\bar{h}\,\bar{\mathbf{v}}_t = \bar{\mathbf{L}}(\bar{\mathbf{u}}) - \boldsymbol{\sigma}\mathbf{n}, \qquad \mathbf{x} \in \Gamma,$$

Matching *accelerations* $\mathbf{v}_t = \bar{\mathbf{v}}_t$ implies

$$\bar{\rho}\bar{h}\,\mathbf{v}_t = \bar{\mathbf{L}}(\bar{\mathbf{u}}) - \boldsymbol{\sigma}\mathbf{n}, \qquad \mathbf{x} \in \Gamma,$$

Using $\rho\mathbf{v}_t = \nabla\cdot\boldsymbol{\sigma}$ and given a predicted value for the beam $\bar{\mathbf{u}}^{(p)}$, the Robin conditions for the fluid are

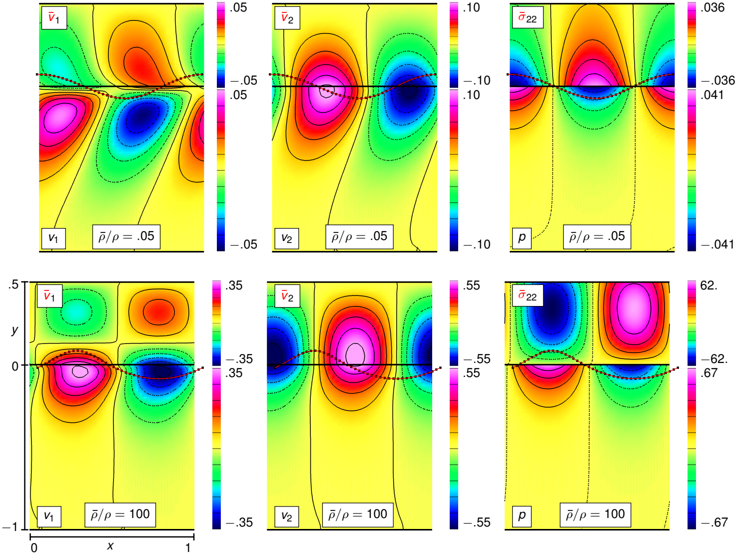
$$\boldsymbol{\sigma}\mathbf{n} + \frac{\bar{\rho}\bar{h}}{\rho}\nabla\cdot\boldsymbol{\sigma} = \bar{\mathbf{L}}\big(\bar{\mathbf{u}}^{(p)}\big), \qquad \mathbf{x} \in \Gamma,$$

Which includes a Robin condition for the pressure

$$\boxed{p + \frac{\bar{\rho}\bar{h}}{\rho}\frac{\partial p}{\partial n}} = \mathbf{n}^T\boldsymbol{\tau}\mathbf{n} + \frac{\mu\bar{\rho}\bar{h}}{\rho}\mathbf{n}^T\Delta\mathbf{v} - \mathbf{n}^T\bar{\mathbf{L}}\big(\bar{\mathbf{u}}^{(p)}\big), \qquad \mathbf{x} \in \Gamma$$

Traveling wave solutions: Stokes + Elastic Solid

Computed solutions for a light and heavy solid match analytic solutions.



Numerical results demonstrate stability and second-order accuracy of the AMP schemes

Confirming normal-mode analysis.

MP-VE, traveling wave, viscous fluid, $\mu = .02$, heavy elastic solid, $\bar{\rho}/\rho = 10^3$										
h_j	$E_j^{(p)}$	r	$E_j^{(\mathbf{v})}$	r	$E_j^{(\bar{\mathbf{u}})}$	r	$E_j^{(\bar{\mathbf{v}})}$	r	$E_j^{(\bar{\boldsymbol{\sigma}})}$	r
1/20	1.2e-2		1.9e-2		2.4e-3		1.6e-2		3.5e1	
1/40	2.9e-3	4.1	3.7e-3	5.1	4.5e-4	5.4	3.1e-3	5.2	9.1e0	3.9
1/80	6.5e-4	4.5	6.0e-4	6.1	8.3e-5	5.4	6.0e-4	5.1	2.5e0	3.6
1/160	1.5e-4	4.3	1.3e-4	4.8	1.6e-5	5.0	1.2e-4	4.8	6.8e-1	3.7
rate	2.18		2.21		2.03		2.03		1.95	

MP-VE, traveling wave, viscous fluid, $\mu = .005$, very light elastic solid, $\bar{\rho}/\rho = 10^{-3}$										
h_j	$E_j^{(p)}$	r	$E_j^{(\mathbf{v})}$	r	$E_j^{(\bar{\mathbf{u}})}$	r	$E_j^{(\bar{\mathbf{v}})}$	r	$E_j^{(\bar{\boldsymbol{\sigma}})}$	r
1/20	2.1e-5		3.2e-4		8.0e-4		2.4e-3		1.3e-5	
1/40	4.6e-6	4.6	9.2e-5	3.4	1.6e-4	4.9	5.8e-4	4.2	3.4e-6	3.9
1/80	9.8e-7	4.7	2.3e-5	4.0	2.7e-5	6.1	10.0e-5	5.8	1.1e-6	3.2
1/160	2.2e-7	4.5	5.7e-6	4.0	4.3e-6	6.3	2.2e-5	4.6	2.9e-7	3.6
rate	2.21		1.93		2.53		2.29		1.82	

Traveling wave solution for a viscous incompressible fluid and elastic solid (MP-VE).

Bulk solids: Traditional scheme is always unstable!

(on sufficiently fine grids, without sub-iterations)

Analysis shows: The Traditional Partitioned (TP) algorithm is formally *unconditionally unstable* (on a fine enough grid).

Theorem

The TP algorithm is stable if and only if $\Delta t \le \frac{2}{\bar{c}_p}\Big(\Delta y - \frac{\rho H}{\bar{\rho}}\Big)$.

2D computations confirm the theory:

MP-VE, traveling wave, TP algorithm				
δy	$\bar{\rho}/\rho = 800$	$\bar{\rho}/\rho = 400$	$\bar{\rho}/\rho = 200$	$\bar{\rho}/\rho = 100$
1/20	stable	stable	stable	stable
1/40	stable	stable	stable	unstable
1/80	stable	stable	unstable	unstable
1/160	stable	unstable	unstable	unstable
1/320	unstable	unstable	unstable	unstable

Summary

1. Traditional partitioned schemes for FSI suffer from an *added-mass* instability for light solids.
2. Traditional schemes can sometimes be stabilized by using multiple sub-iterations per time step, or through the use of Robin (mixed) interface conditions (which still generally require sub-iterations).
3. We have developed AMP schemes that over-come the added-mass instability for a variety of regimes and *require no sub-iterations*.
4. AMP schemes for incompressible flows and compressible elastic bulk solids or elastic shells/beams were described and shown to be stable without iterations even for light solids.

The Overture software is freely available from `overtureFramework.org` (does not yet include beam models).