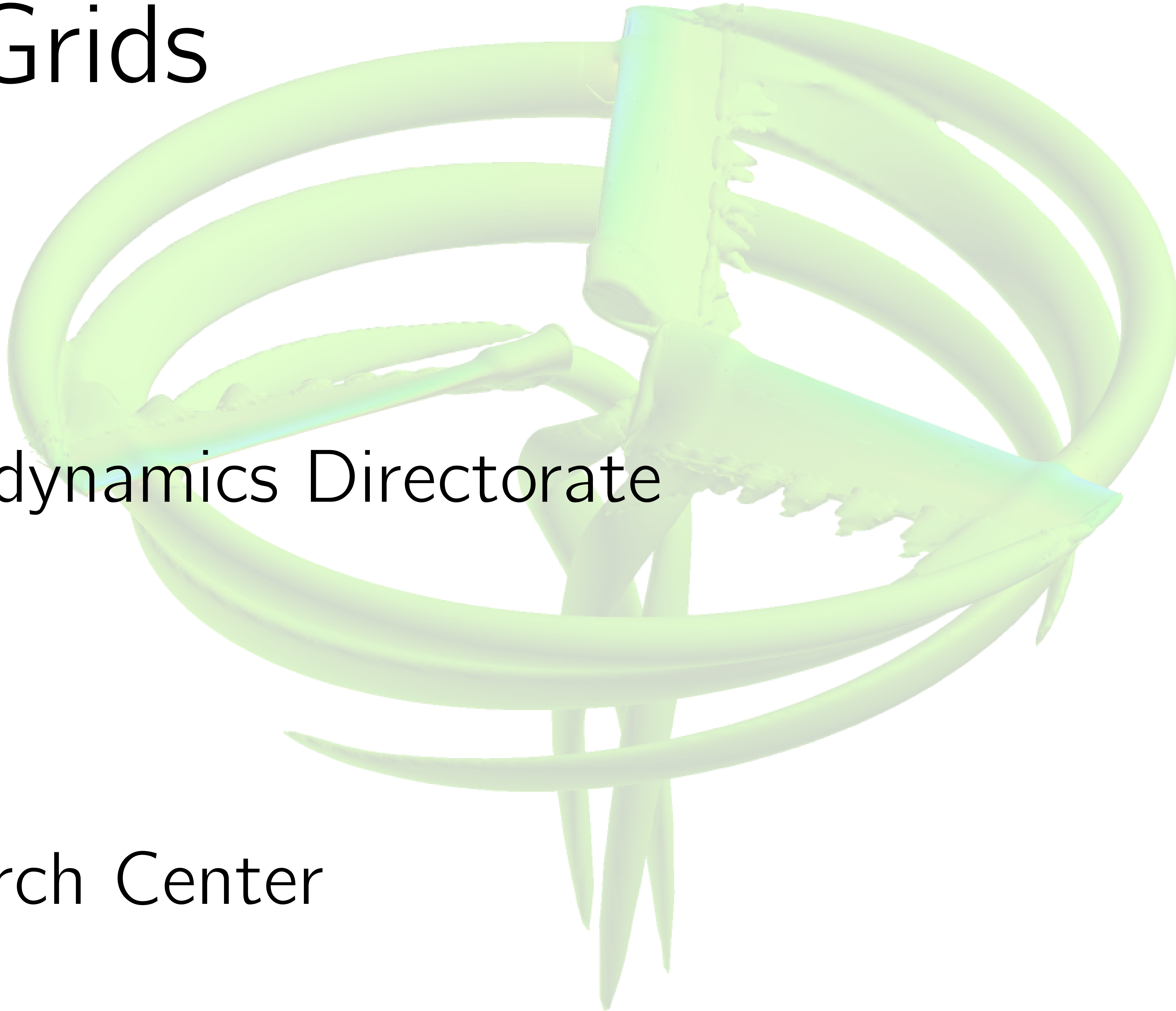
A Time-Spectral Method for Relative Motion on Overset Grids



Joshua Leffell
Stanford University
US Army Aeroflightdynamics Directorate

Scott M. Murman
Thomas H. Pulliam
NASA Ames Research Center

12th Symposium on Overset Composite Grids, Georgia Institute of Technology, October 9, 2014

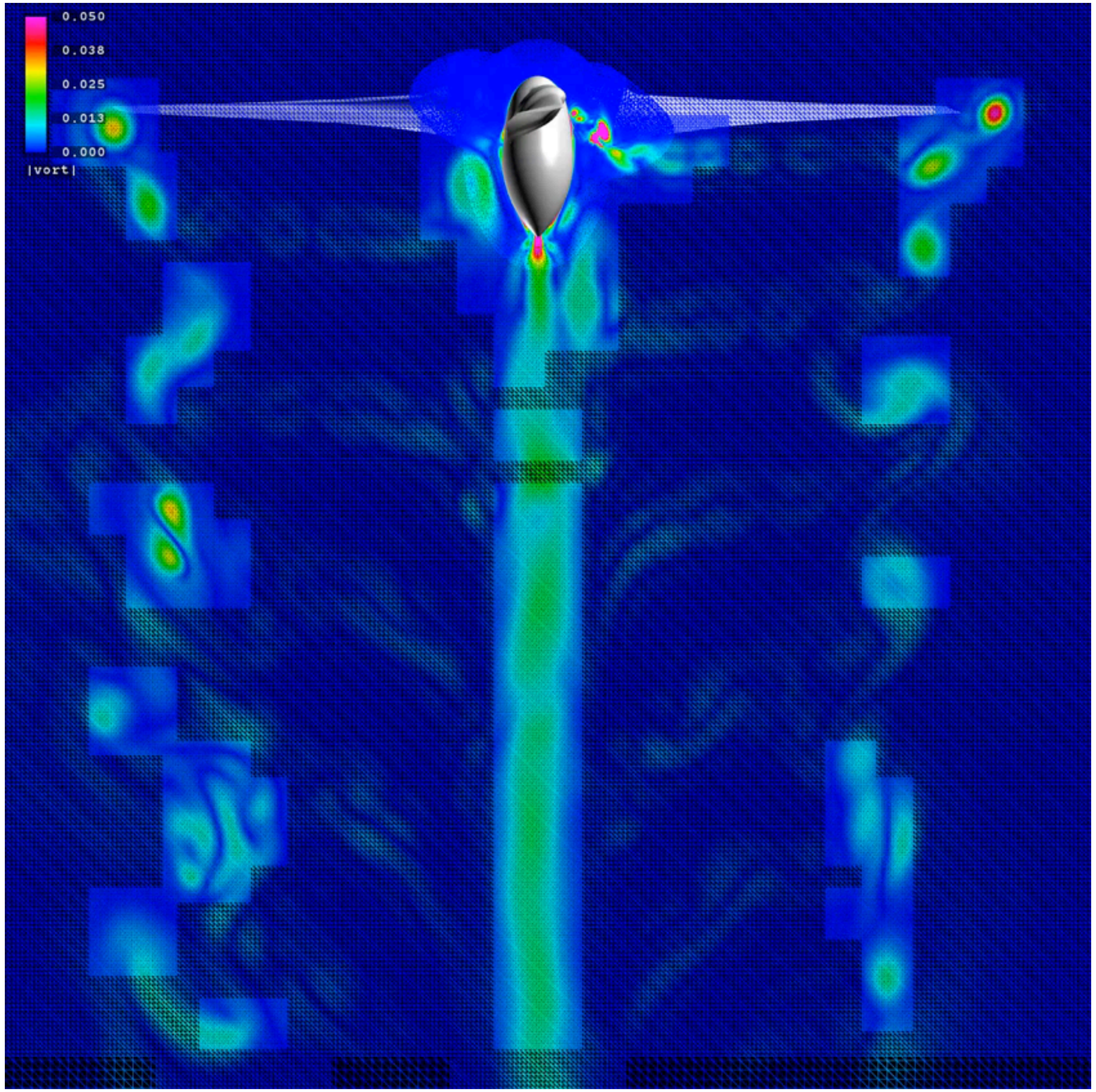
Motivation

Overset Grid Technology (OVERFLOW)

- Complex geometry
- Efficient data structures
- Versatile for moving-body configurations

Time-Spectral Method

- Periodic flow $\longrightarrow$ steady in frequency domain
- Avoid time-accurate transient
- Few temporal DOF



V22 in Hover, Neal Chaderjian, NASA Ames Research Center

Motivation

Overset Grid Technology (OVERFLOW)

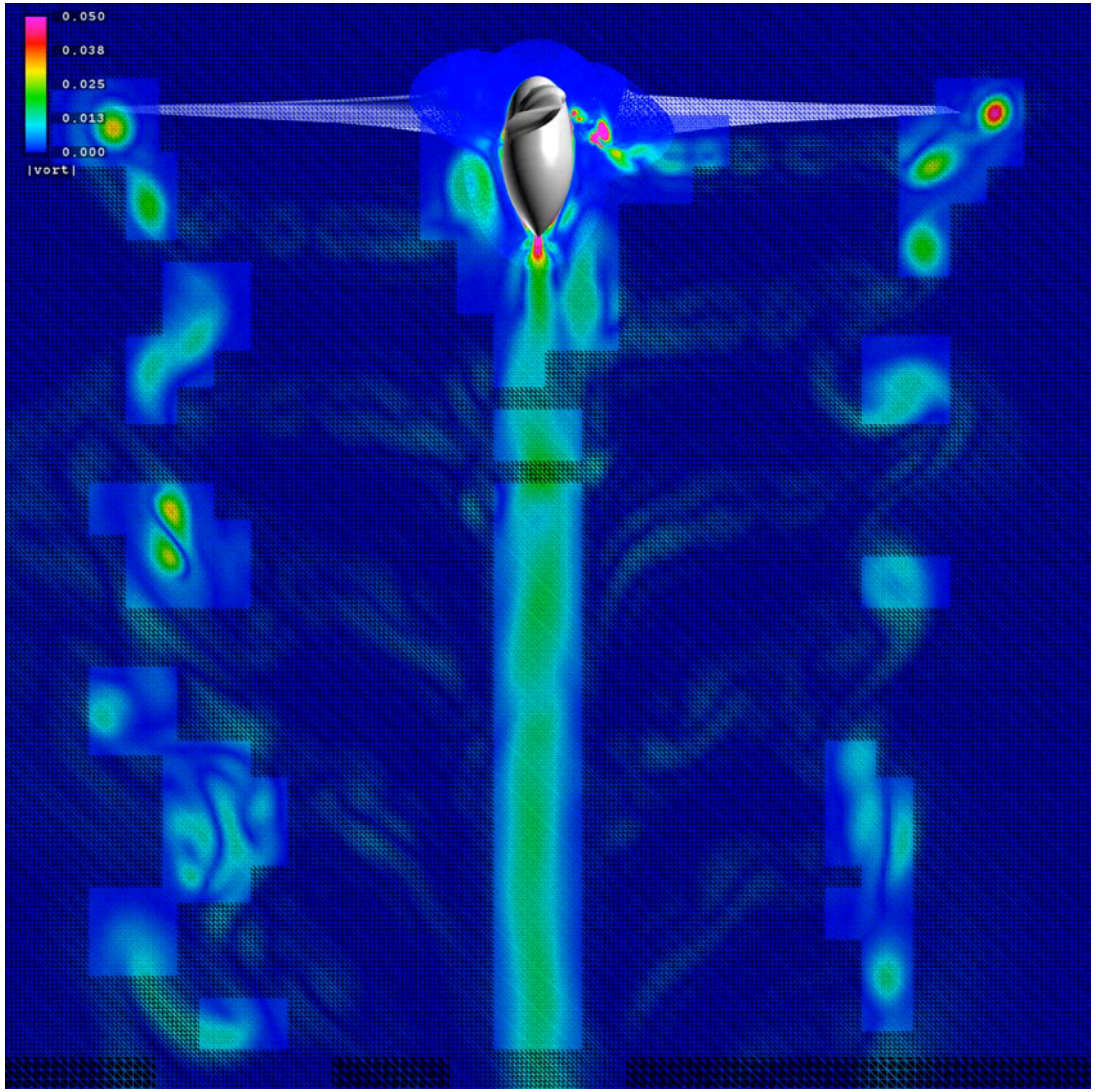
- Complex geometry
- Efficient data structures
- Versatile for moving-body configurations

Time-Spectral Method

- Periodic flow $\longrightarrow$ steady in frequency domain
- Avoid time-accurate transient
- Few temporal DOF

Objective

Merge the two techniques in a general and consistent manner



V22 in Hover, Neal Chaderjian, NASA Ames Research Center

Outline

The Time-Spectral Method

Hybrid Time-Spectral Scheme

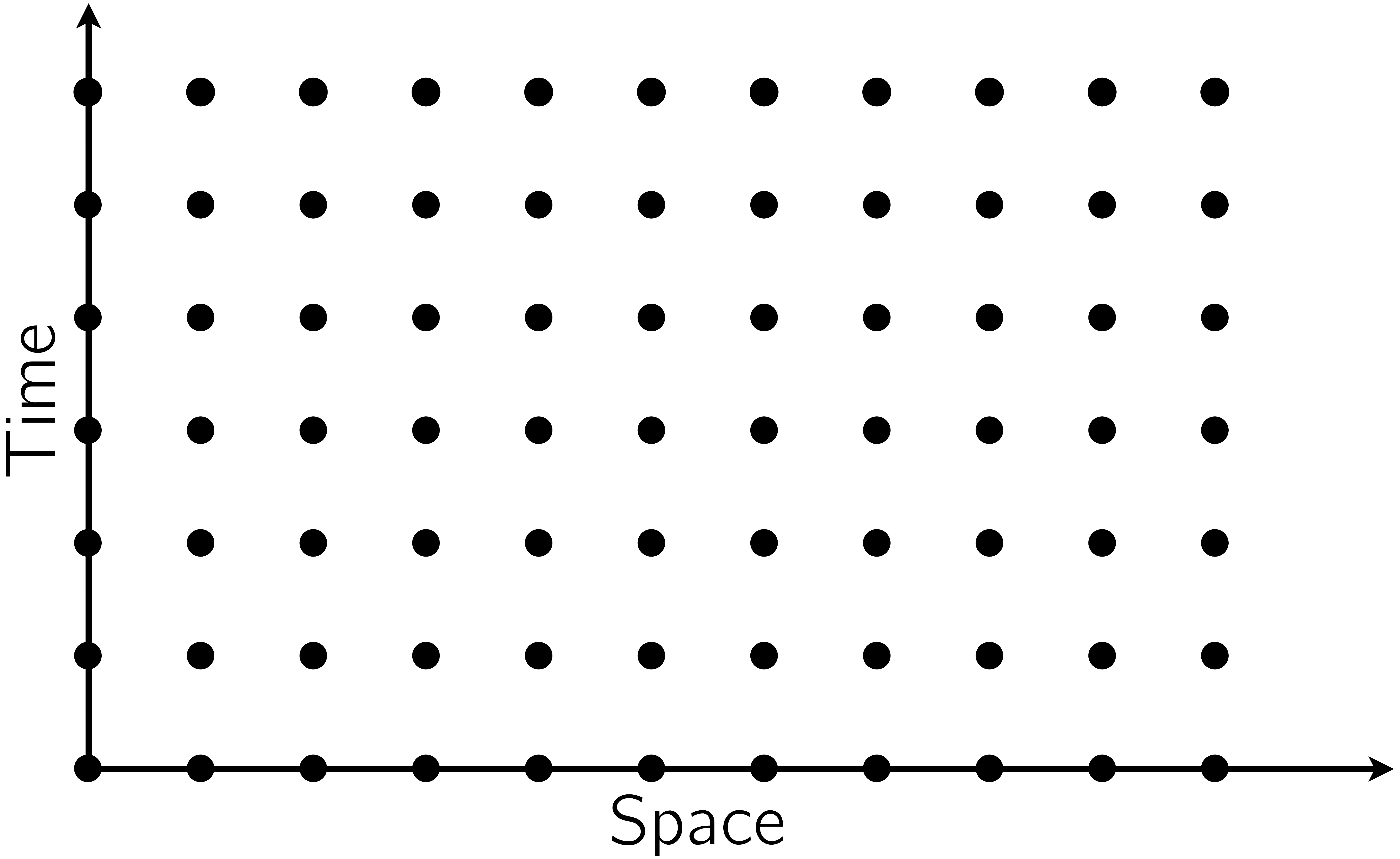
OVERFLOW

Numerical Results

Summary & Future Work

The Time-Spectral Method

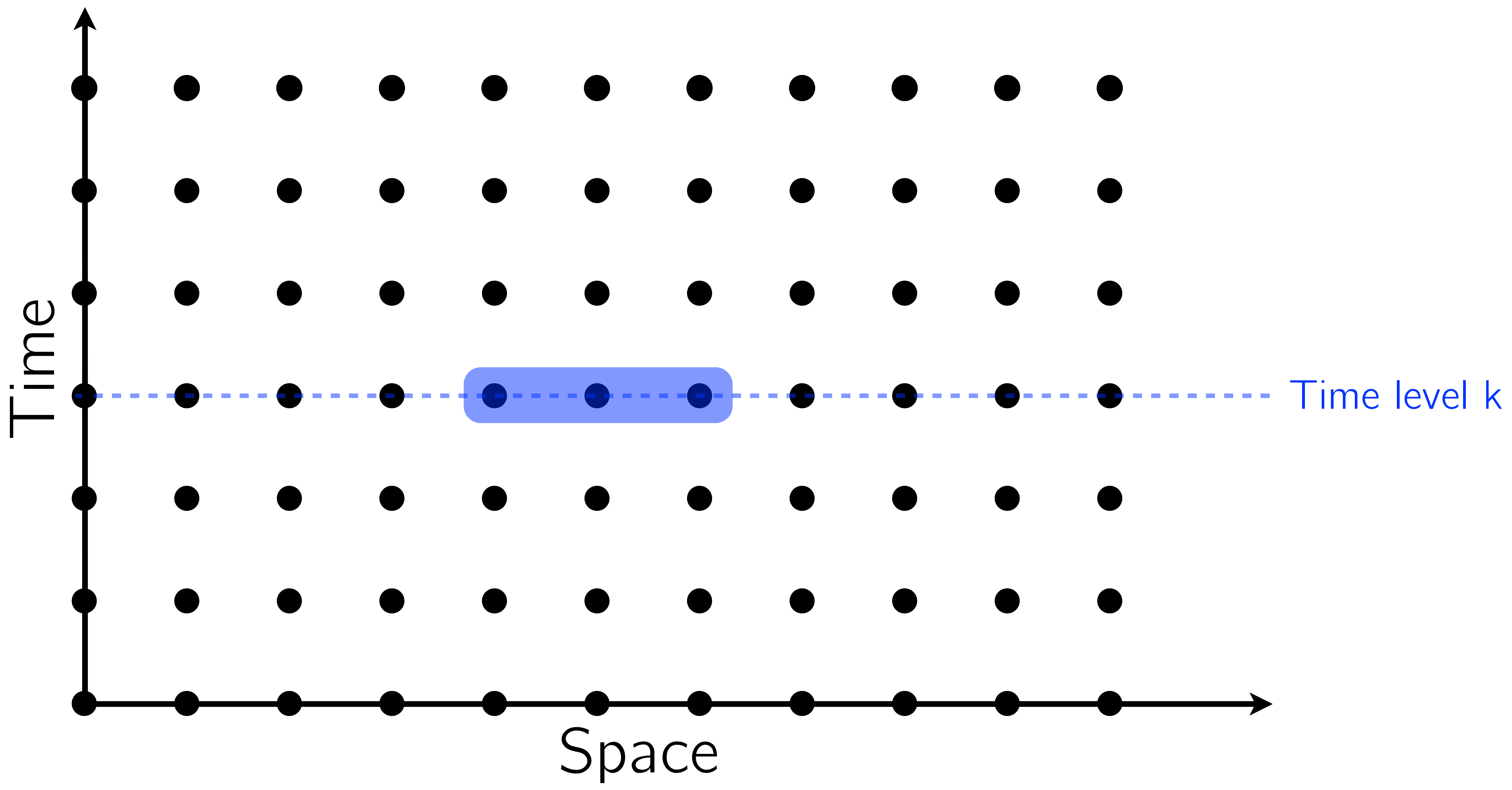
Space-Time Domain



The Time-Spectral Method

Space-Time Domain

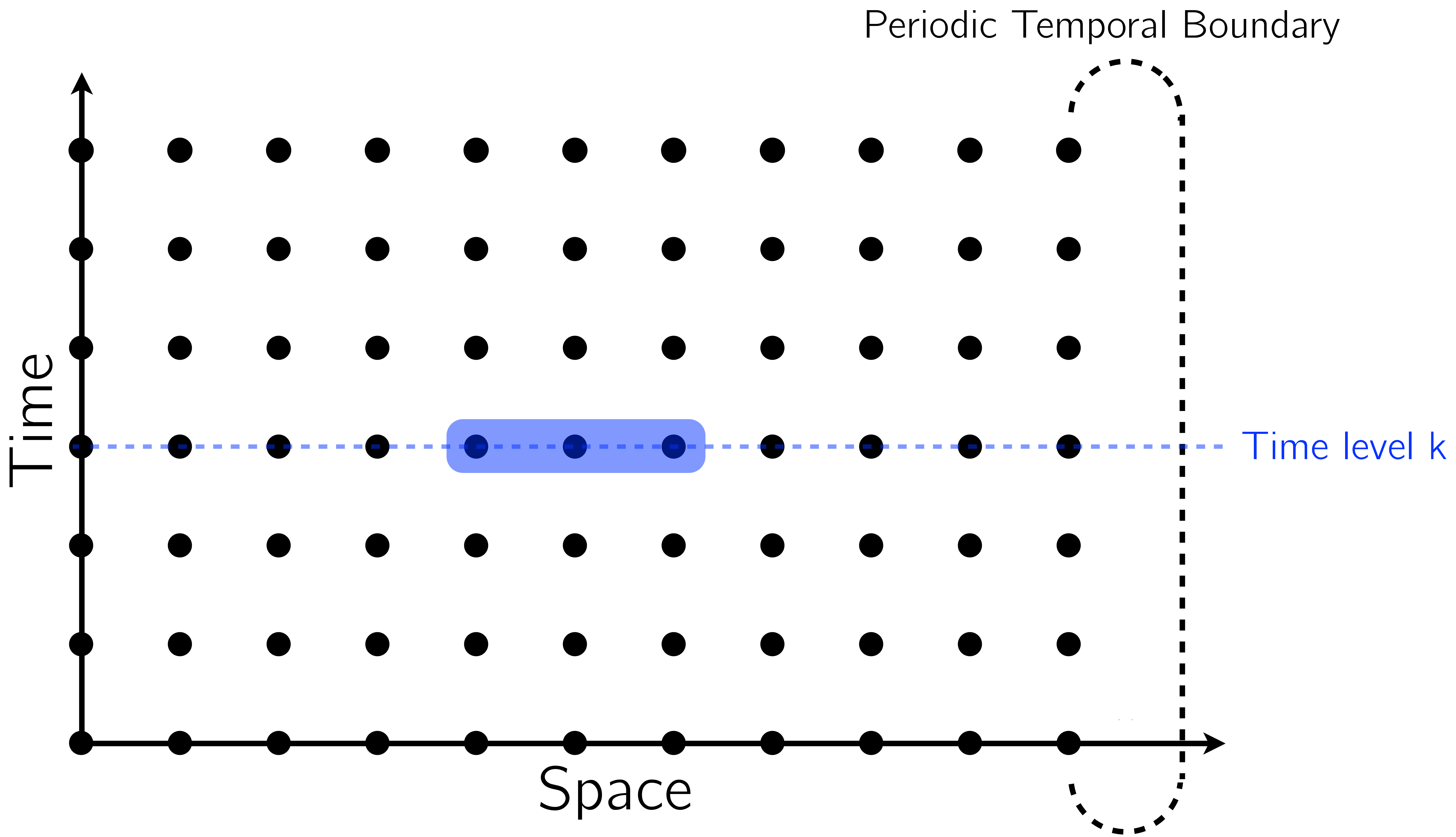
Local support in space (finite differences)



The Time-Spectral Method

Space-Time Domain

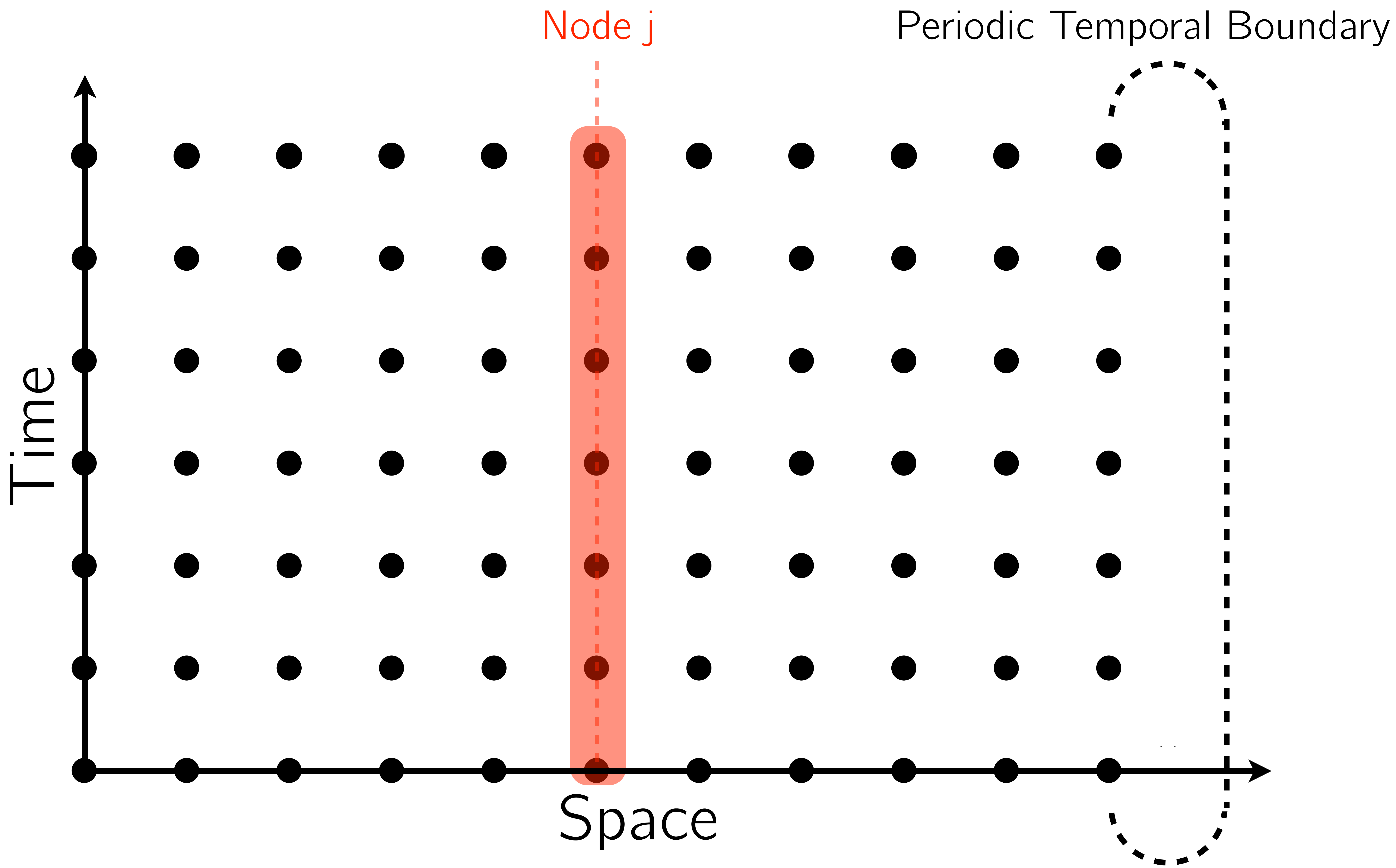
Local support in space (finite differences)



The Time-Spectral Method

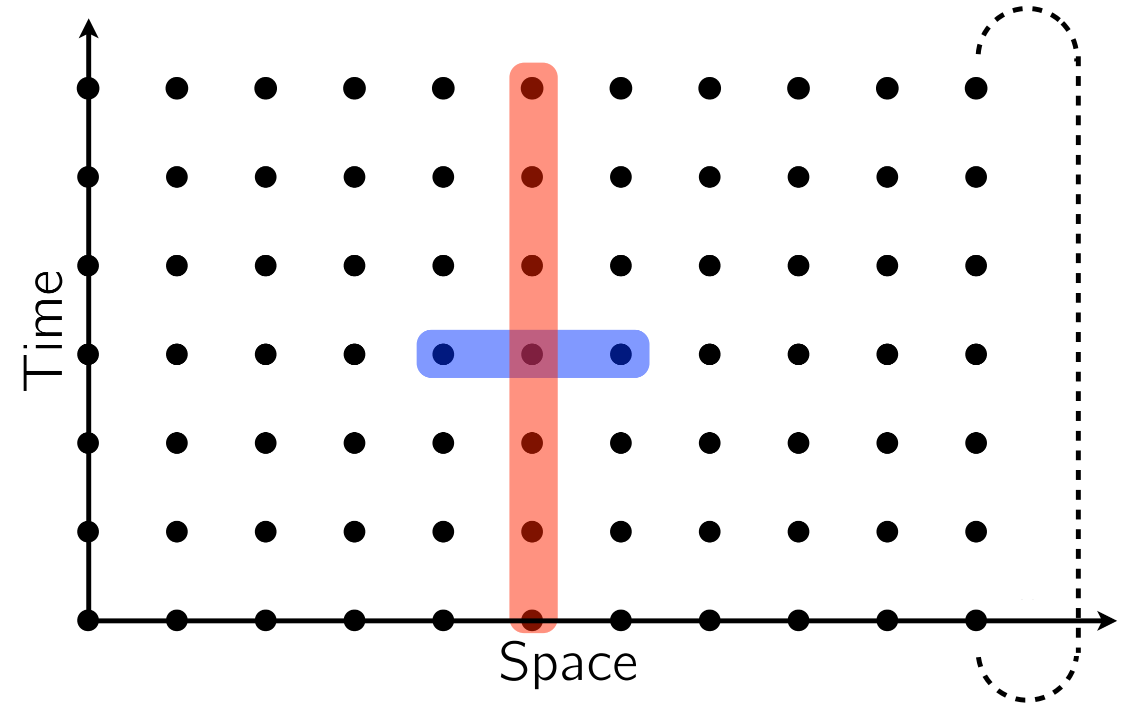
Space-Time Domain

Infinite support in time (Fourier series)



The Time-Spectral Method

Fourier Collocation in Time - Problem Statement



Time-periodic PDE

$$\frac{\partial}{\partial t} u(\mathbf{x}, t) + \mathcal{R}(u(\mathbf{x}, t)) = 0, \quad \mathbf{x} \in \Omega, \quad \text{s.t. } u(\mathbf{x}, t+T) = u(\mathbf{x}, t)$$

Fourier series

$$u(\mathbf{x}, t) = \sum_{k=-\infty}^{\infty} \tilde{u}_k(\mathbf{x}) \phi_k(t), \quad \phi_k(t) = e^{i\omega k t} \quad \longrightarrow \quad u_N(\mathbf{x}, t) = \sum_{k=-K}^{K} \tilde{u}_k(\mathbf{x}) \phi_k(t)$$

Method of weighted residuals

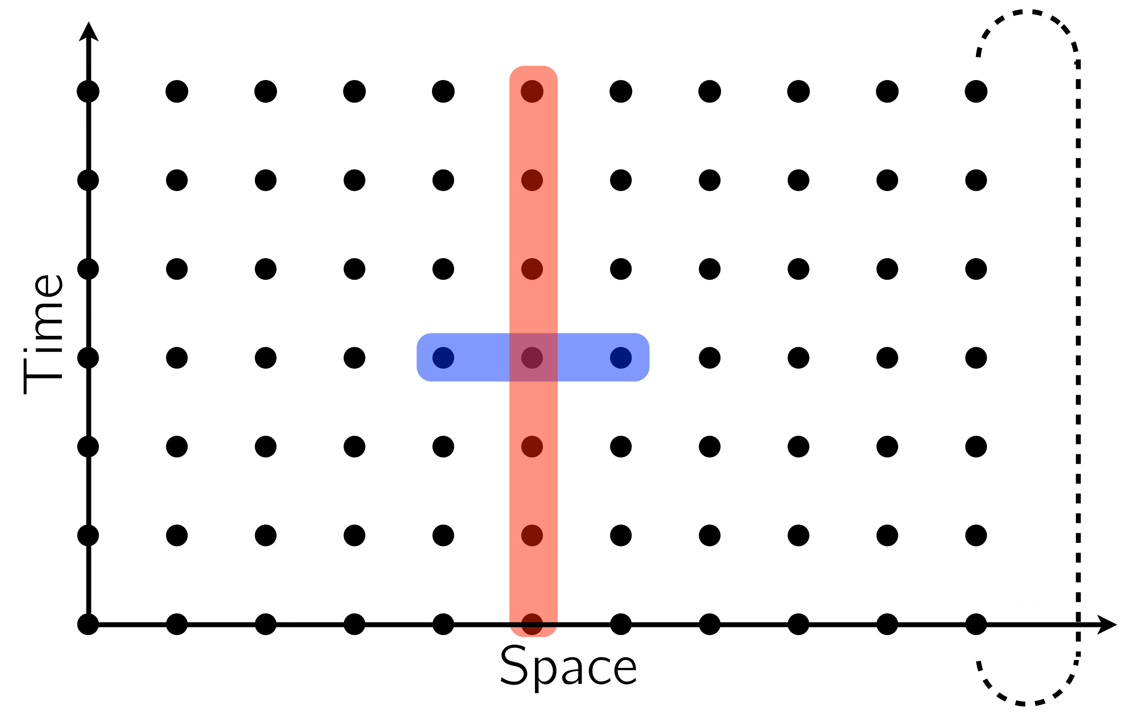
$$(R_N, \psi_j)_w = \int_0^T w R_N(\mathbf{x}, t) \psi_j \, dt = 0, \quad \mathbf{x} \in \Omega, \quad j \in \{0, \ldots, N-1\}$$

$$\psi_j = \delta(t - t_j), \quad w = 1, \quad t_j = \frac{jT}{N}$$

$$R_N(\mathbf{x}, t) = \frac{\partial}{\partial t} u_N(\mathbf{x}, t) + \mathcal{R}(u_N(\mathbf{x}, t)), \quad \mathbf{x} \in \Omega$$

The Time-Spectral Method

Fourier Collocation in Time - Problem Statement



Time-periodic PDE

$$\frac{\partial}{\partial t} u(\mathbf{x}, t) + \mathcal{R}(u(\mathbf{x}, t)) = 0, \quad \mathbf{x} \in \Omega, \quad \text{s.t. } u(\mathbf{x}, t+T) = u(\mathbf{x}, t)$$

Fourier series

$$u(\mathbf{x}, t) = \sum_{k=-\infty}^{\infty} \tilde{u}_k(\mathbf{x}) \phi_k(t), \quad \phi_k(t) = e^{i\omega k t} \longrightarrow u_N(\mathbf{x}, t) = \sum_{k=-K}^{K} \tilde{u}_k(\mathbf{x}) \phi_k(t)$$

K modes $\longrightarrow$ N = 2K + 1 Samples

Method of weighted residuals

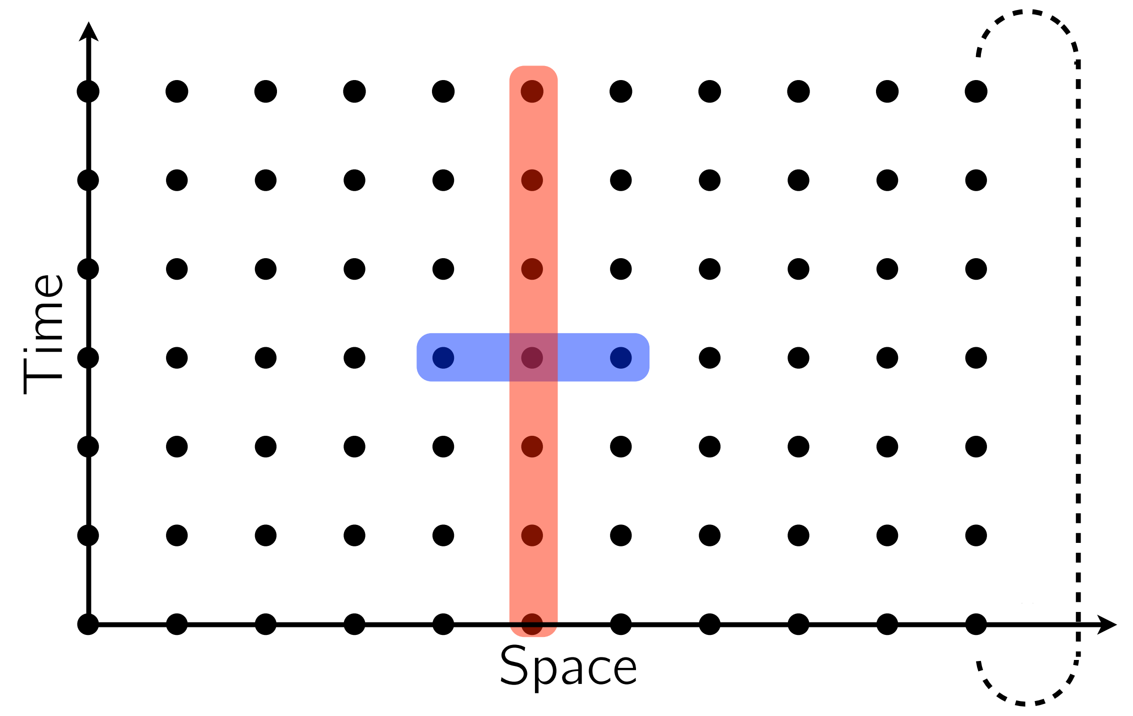
$$(R_N, \psi_j)_w = \int_0^T w R_N(\mathbf{x}, t) \psi_j \, dt = 0, \quad \mathbf{x} \in \Omega, \quad j \in \{0, \ldots, N-1\}$$

$$\psi_j = \delta(t - t_j), \quad w = 1, \quad t_j = \frac{jT}{N}$$

$$R_N(\mathbf{x}, t) = \frac{\partial}{\partial t} u_N(\mathbf{x}, t) + \mathcal{R}(u_N(\mathbf{x}, t)), \quad \mathbf{x} \in \Omega$$

The Time-Spectral Method

Fourier Collocation in Time - Problem Statement



Time-periodic PDE

$$\frac{\partial}{\partial t}u(\mathbf{x},t)+\mathcal{R}(u(\mathbf{x},t))=0, \quad \mathbf{x}\in\Omega, \quad \text{s.t. } u(\mathbf{x},t+T)=u(\mathbf{x},t)$$

Fourier series

$$u(\mathbf{x},t)=\sum_{k=-\infty}^{\infty}\tilde{u}_k(\mathbf{x})\,\phi_k(t), \quad \phi_k(t)=e^{i\omega kt} \longrightarrow u_N(\mathbf{x},t)=\sum_{k=-K}^{K}\tilde{u}_k(\mathbf{x})\,\phi_k(t)$$

K modes $\longrightarrow$ N = 2K + 1 Samples

Method of weighted residuals

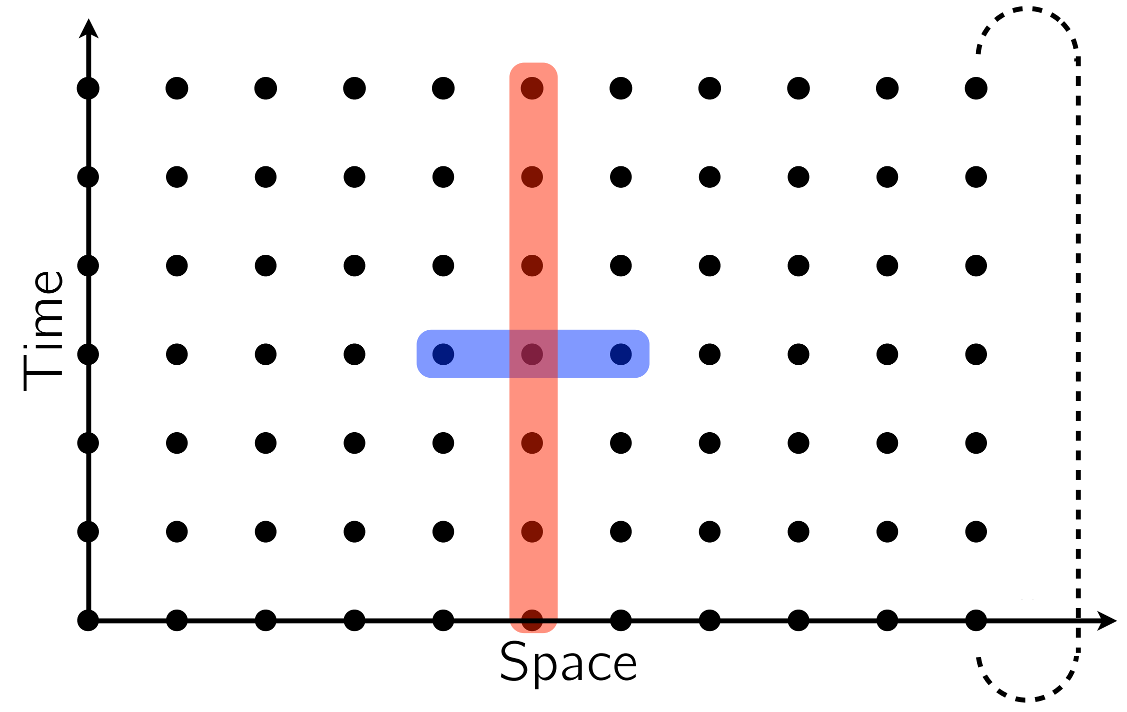
$$(R_N,\psi_j)_w=\int_0^T wR_N(\mathbf{x},t)\,\psi_j\,dt=0, \quad \mathbf{x}\in\Omega, \quad j\in\{0,\ldots,N-1\}$$

$$\psi_j=\delta(t-t_j), \quad w=1, \quad t_j=\frac{jT}{N}$$

$$R_N(\mathbf{x},t_j)=\frac{\partial}{\partial t}u_N(\mathbf{x},t_j)+\mathcal{R}(u_N(\mathbf{x},t_j))=0, \quad \mathbf{x}\in\Omega, \quad j\in\{0,\ldots,N-1\}$$

The Time-Spectral Method

Fourier Collocation in Time - Differentiation Operator



Analytically differentiate Fourier series

$$u_N(\mathbf{x},t) = \sum_{k=-K}^{K} \tilde{u}_k(\mathbf{x})\, e^{i\omega k t} \longrightarrow \frac{\partial}{\partial t} u_N(\mathbf{x},t) = \sum_{k=-K}^{K} i\omega k \tilde{u}_k(\mathbf{x})\, e^{i\omega k t}$$

Discrete Fourier Transform

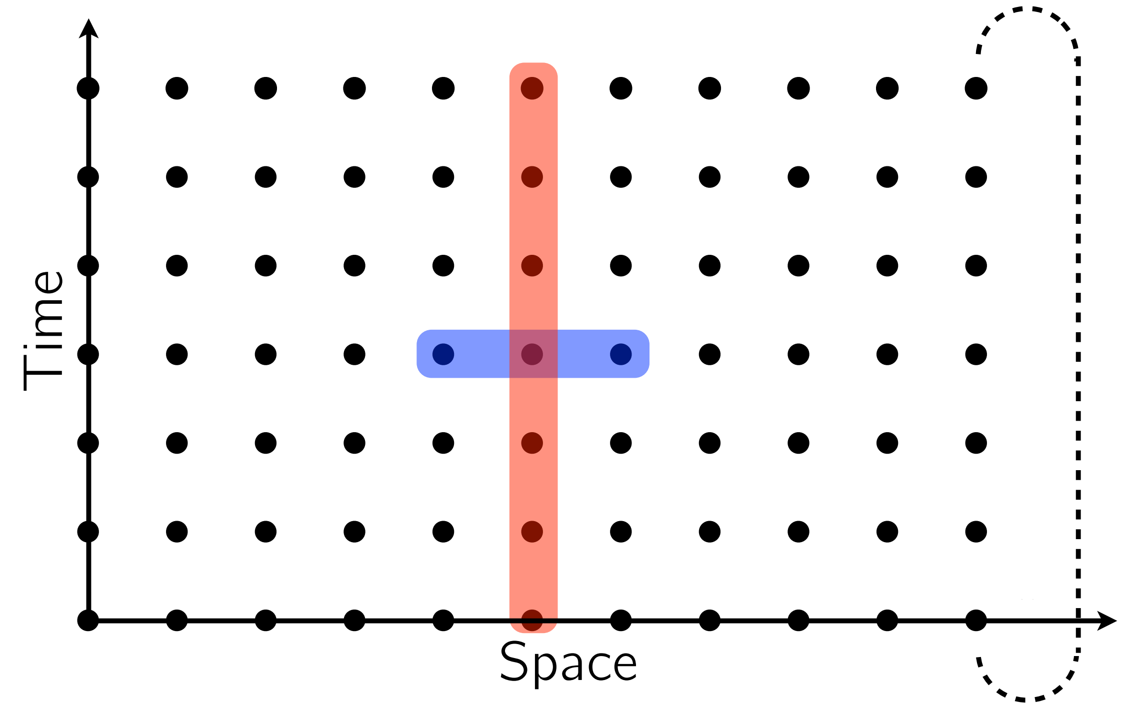
$$\tilde{u}_k(\mathbf{x}) = \frac{1}{N} \sum_{j=0}^{N-1} u_N(\mathbf{x}, t_j)\, e^{-i\omega k t_j}$$

Fourier interpolation differentiation operator

$$\begin{aligned} \frac{\partial}{\partial t} u_N(\mathbf{x}, t_j) &= \sum_{n=0}^{N-1} u_N(\mathbf{x}, t_n) \sum_{k=-K}^{K} \frac{i\omega k}{N} e^{i\omega k (t_j - t_n)} \\ &= \sum_{n=0}^{N-1} d_{jn} u_N(\mathbf{x}, t_n) \end{aligned}$$

The Time-Spectral Method

Fourier Collocation in Time - Differentiation Operator



Analytically differentiate Fourier series

$$u_N(\mathbf{x},t) = \sum_{k=-K}^{K} \tilde{u}_k(\mathbf{x}) e^{i\omega kt} \longrightarrow \frac{\partial}{\partial t} u_N(\mathbf{x},t) = \sum_{k=-K}^{K} i\omega k \tilde{u}_k(\mathbf{x}) e^{i\omega kt}$$

Discrete Fourier Transform

$$\tilde{u}_k(\mathbf{x}) = \frac{1}{N} \sum_{j=0}^{N-1} u_N(\mathbf{x},t_j) e^{-i\omega k t_j}$$

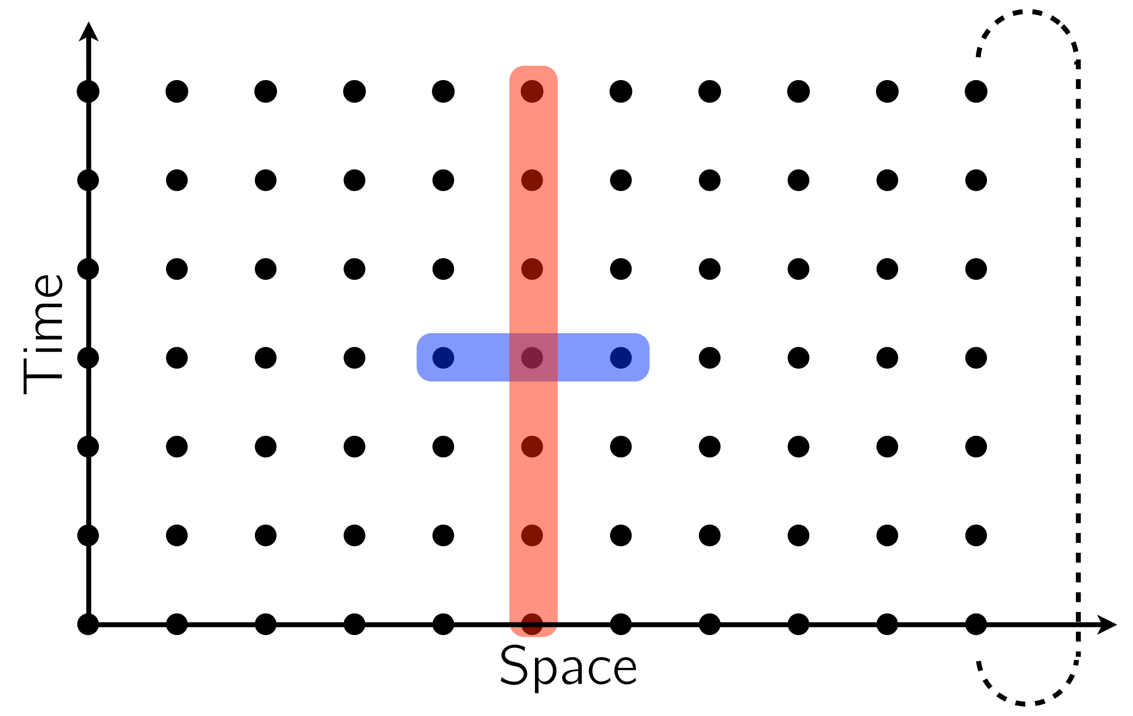
Fourier interpolation differentiation operator

$$\begin{aligned} \frac{\partial}{\partial t} u_N(\mathbf{x},t_j) &= \sum_{n=0}^{N-1} u_N(\mathbf{x},t_n) \sum_{k=-K}^{K} \frac{i\omega k}{N} e^{i\omega k(t_j - t_n)} \\ &= \sum_{n=0}^{N-1} d_{jn} u_N(\mathbf{x},t_n) \end{aligned}$$

$$\boxed{\frac{\partial}{\partial t} \mathbf{u}_N(\mathbf{x}) = \mathcal{D}_N \mathbf{u}_N(\mathbf{x})} \qquad \mathbf{u}_N(\mathbf{x}) = \{u_N(\mathbf{x},t_0), \ldots, u_N(\mathbf{x},t_{N-1})\}^T$$

The Time-Spectral Method

Fourier Collocation in Time



Semi-discrete form

$$\frac{\partial}{\partial t}\mathbf{u}_N\left(\mathbf{x}\right) + \mathcal{R}\left(\mathbf{u}_N\left(\mathbf{x}\right)\right) = 0, \quad \mathbf{x} \in \Omega$$

Fully discrete form

$$\mathcal{D}_N\mathbf{u}_N\left(\mathbf{x}\right) + \mathcal{R}\left(\mathbf{u}_N\left(\mathbf{x}\right)\right) = 0, \quad \mathbf{x} \in \Omega$$

Pseudotime integration

$$\boxed{\frac{\partial}{\partial \tau}\mathbf{u}_N\left(\mathbf{x}\right) + \mathcal{D}_N\mathbf{u}_N\left(\mathbf{x}\right) + \mathcal{R}\left(\mathbf{u}_N\left(\mathbf{x}\right)\right) = 0, \quad \mathbf{x} \in \Omega}$$

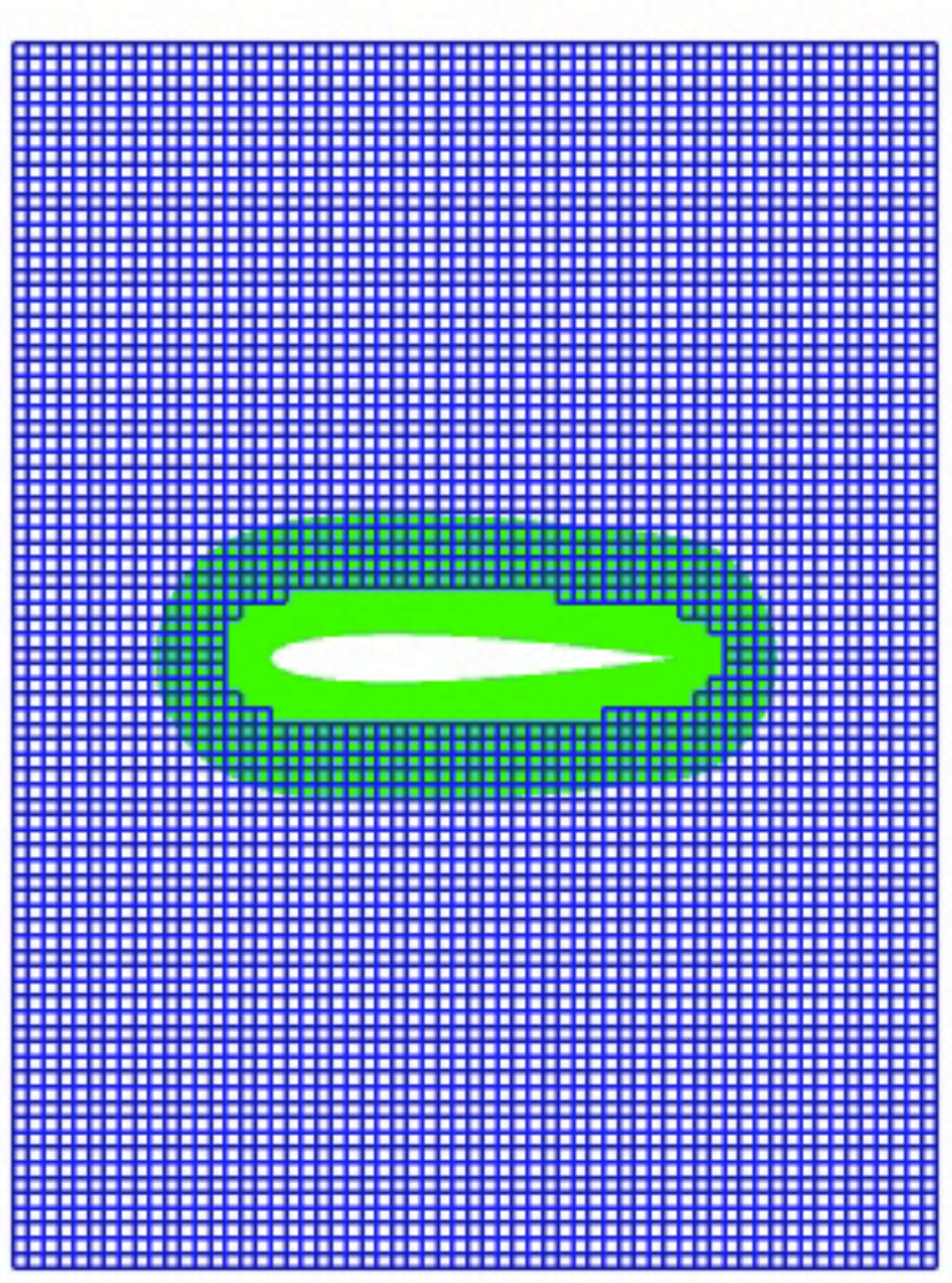
Interpolation/Reconstruction Postprocessing

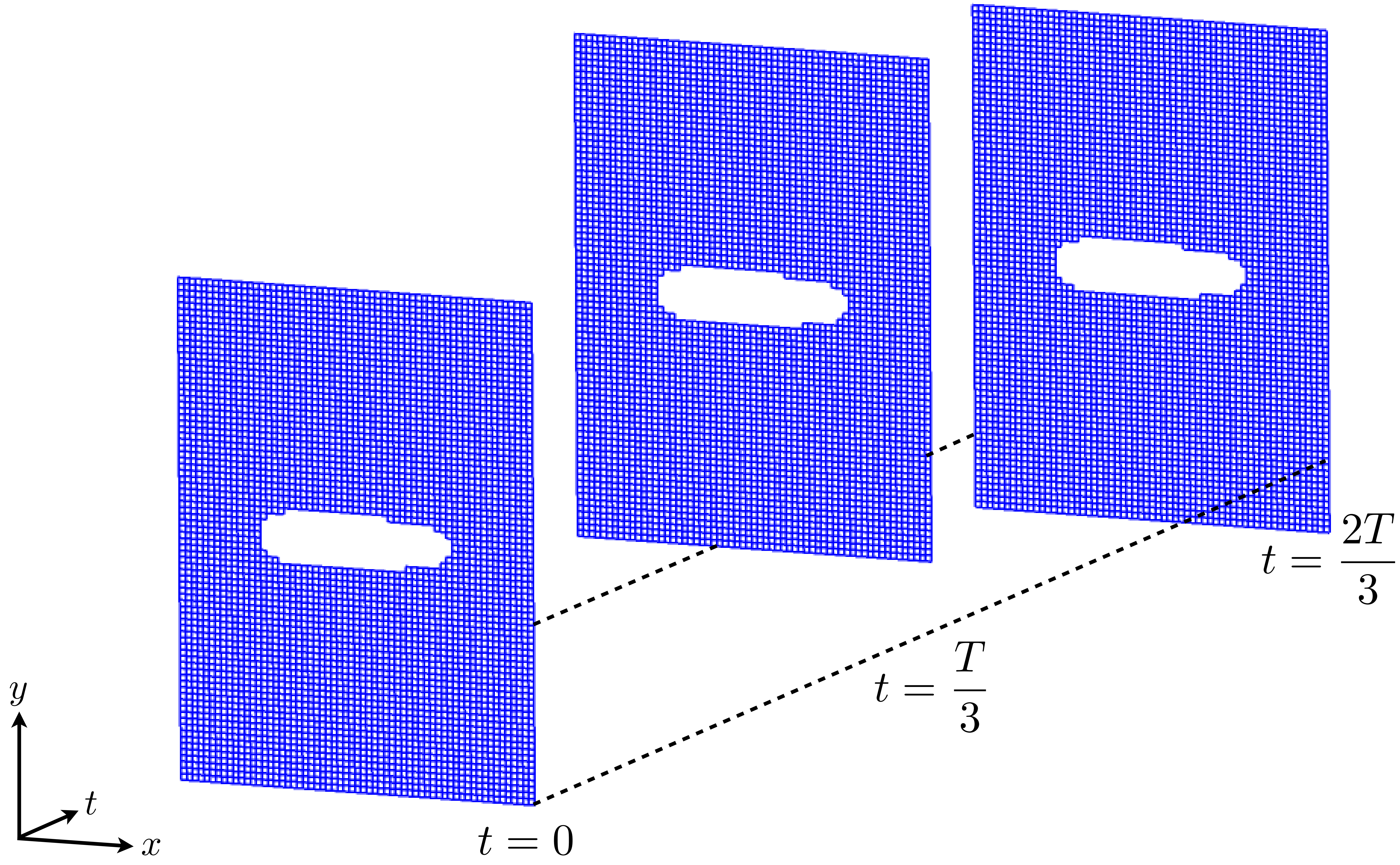
$$\tilde{u}_k\left(\mathbf{x}\right) = \frac{1}{N}\sum_{j=0}^{N-1} u_N\left(\mathbf{x}, t_j\right) e^{-i\omega k t_j} \longrightarrow u_N\left(\mathbf{x}, t\right) = \sum_{k=-K}^{K} \tilde{u}_k\left(\mathbf{x}\right)\phi_k\left(t\right)$$

Outline

Hybrid Time-Spectral Method

Rigid Motion



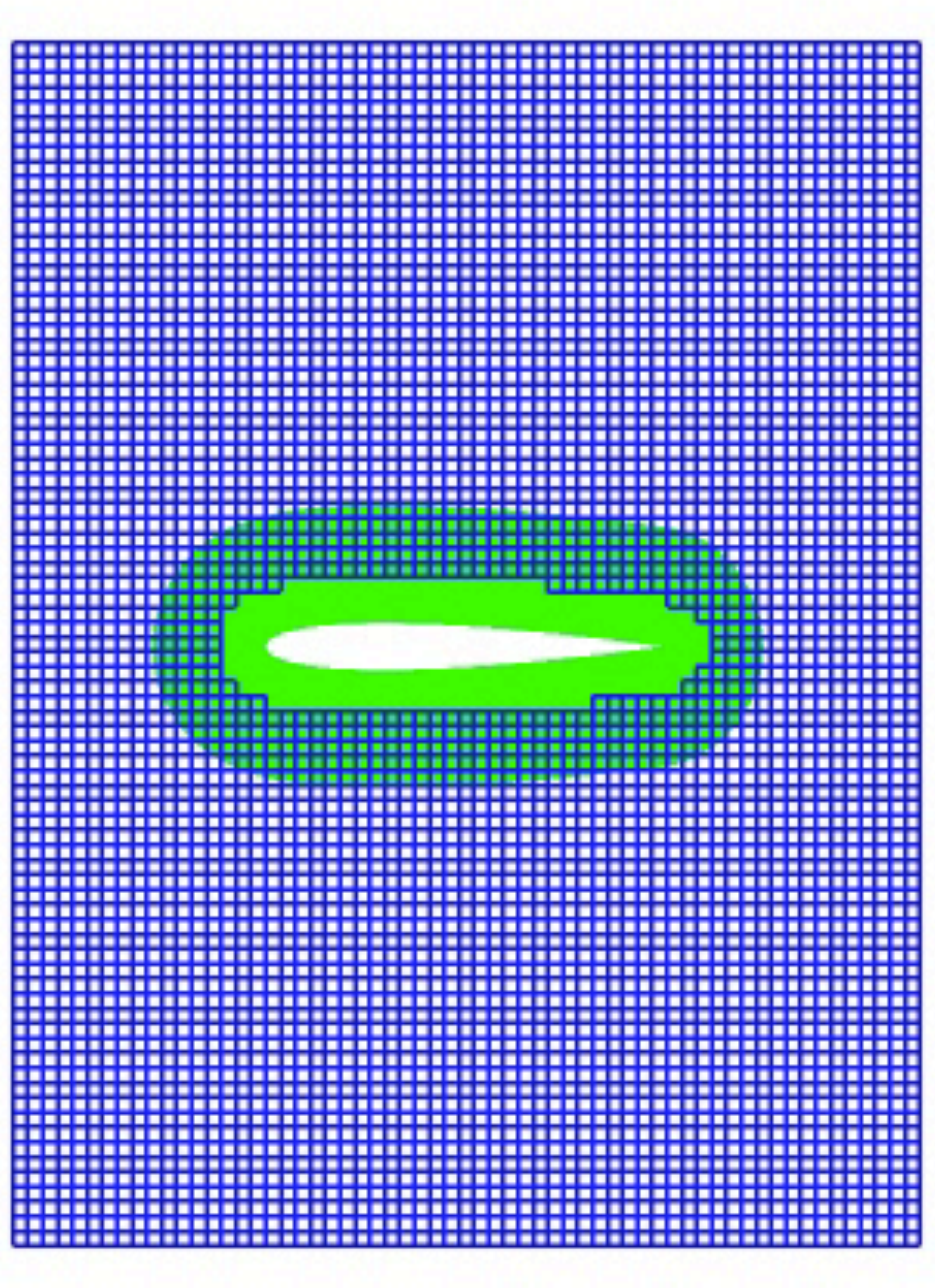


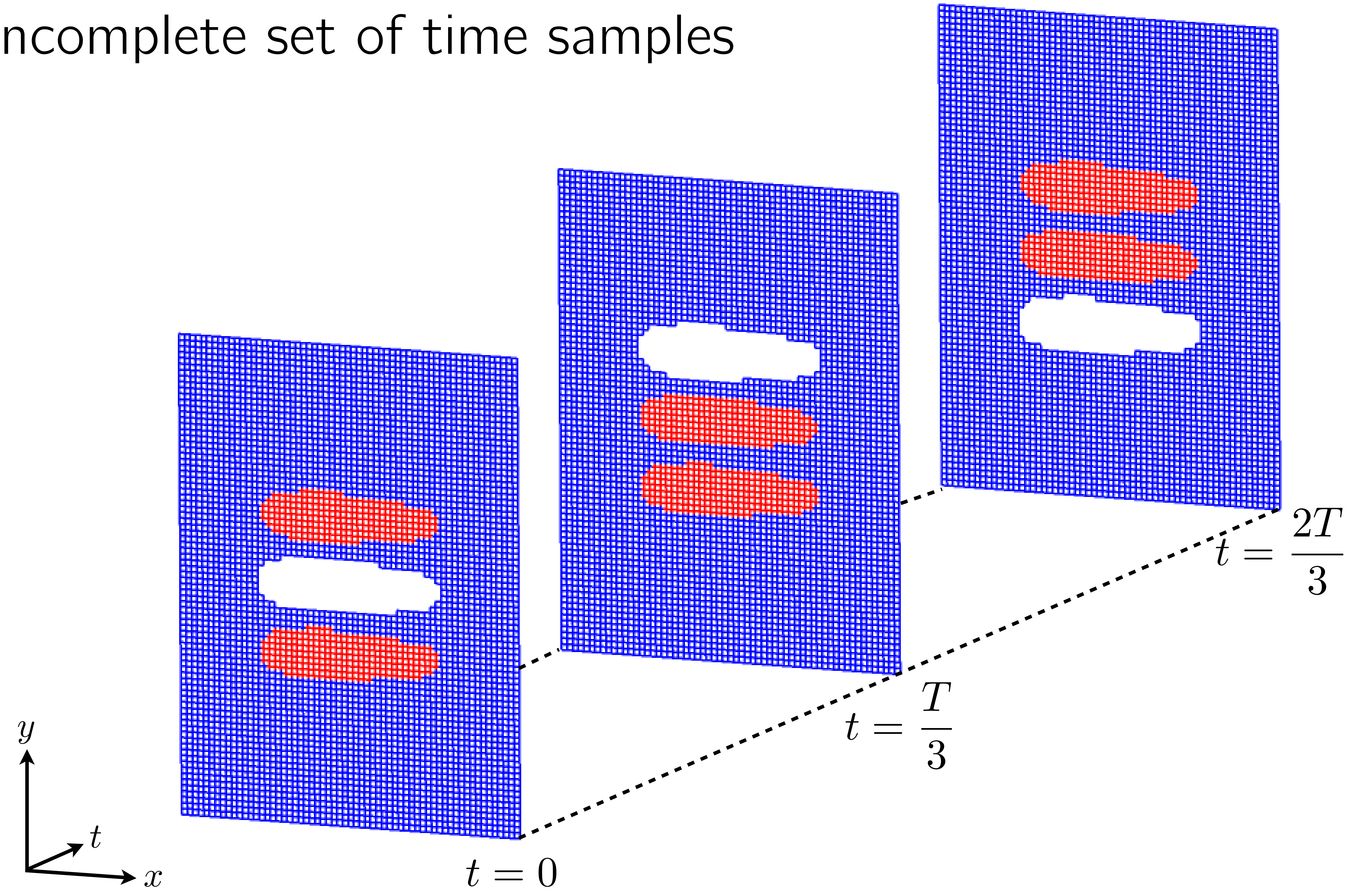
Constant blanking

Hybrid Time-Spectral Method

Relative Motion

Points in hole-cut regions have an incomplete set of time samples

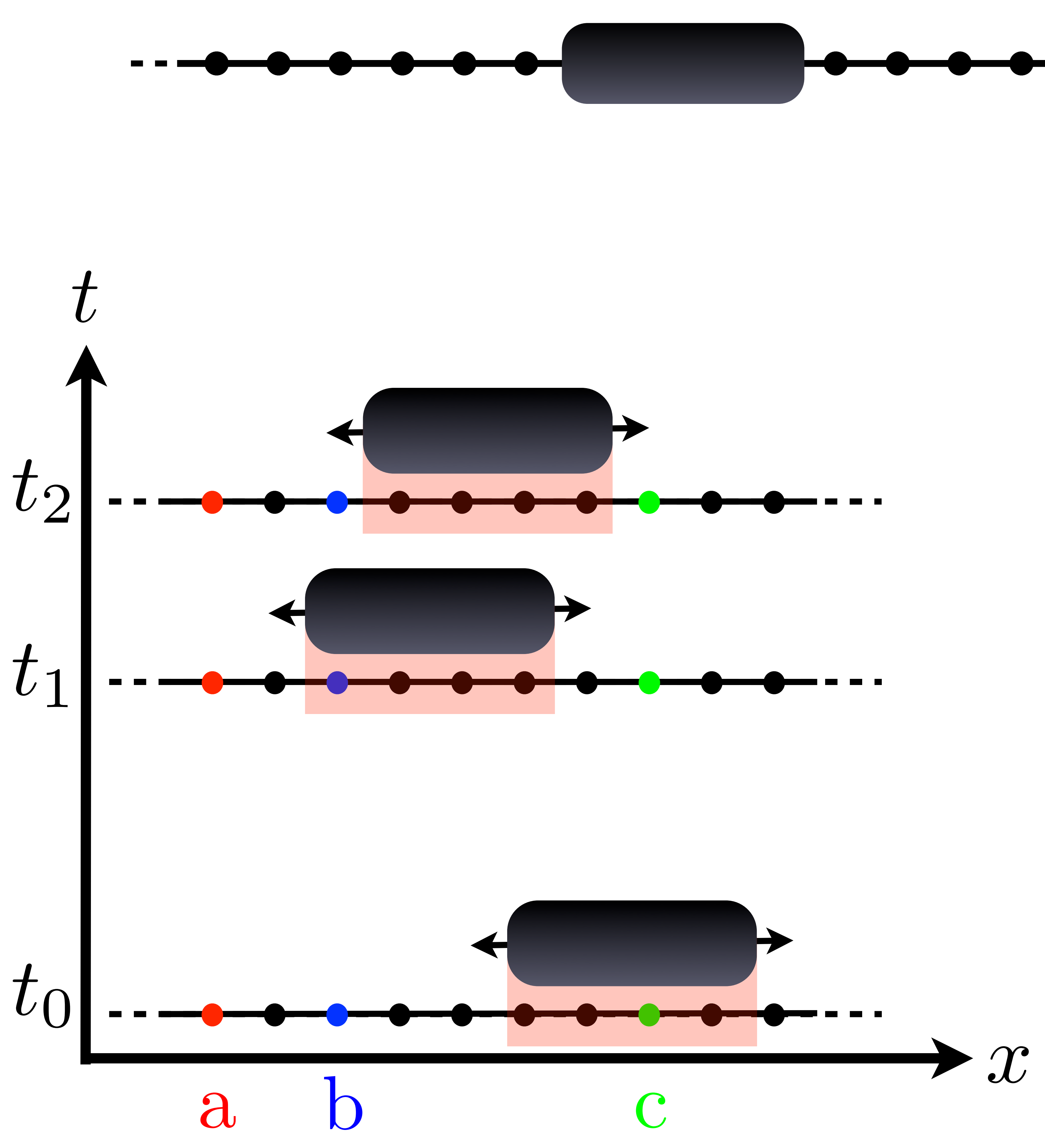




Dynamic blanking

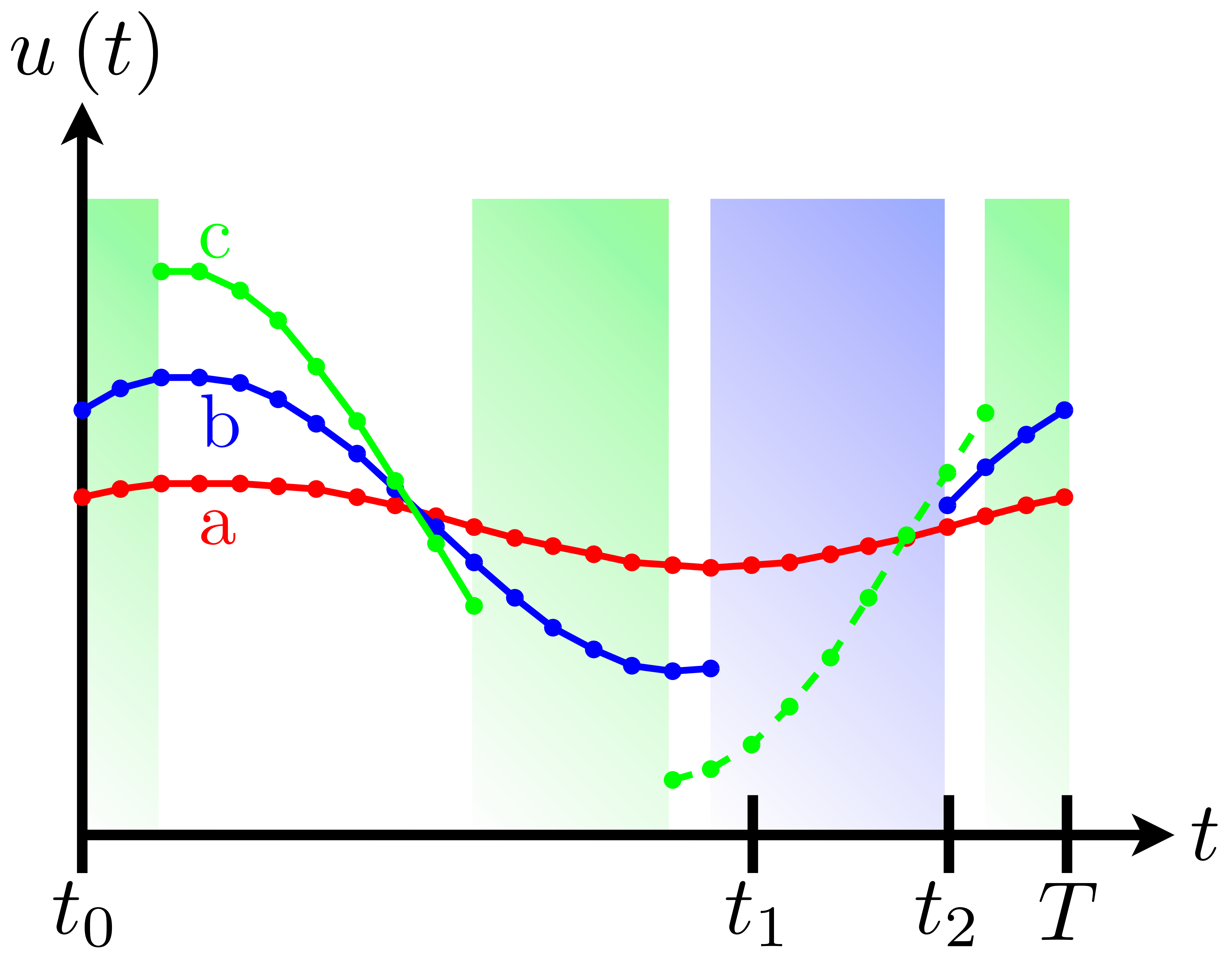
Hybrid Time-Spectral Method

Proposed Approach



Oscillating Piston Example

- Node a never covered by piston
- Node b covered once per period
- Node c covered twice per period



∗ Leffell, J. I., Murman, S. M., and Pulliam, T. H. , "An Extension of the Time-Spectral Method to Overset Solvers," AIAA Paper 0637, Grapevine, Texas, January 2013
Leffell, J. I., "An Overset Time-Spectral Method for Relative Motion", PhD Thesis, Stanford University, June 2014

Hybrid Time-Spectral Method

Proposed Approach

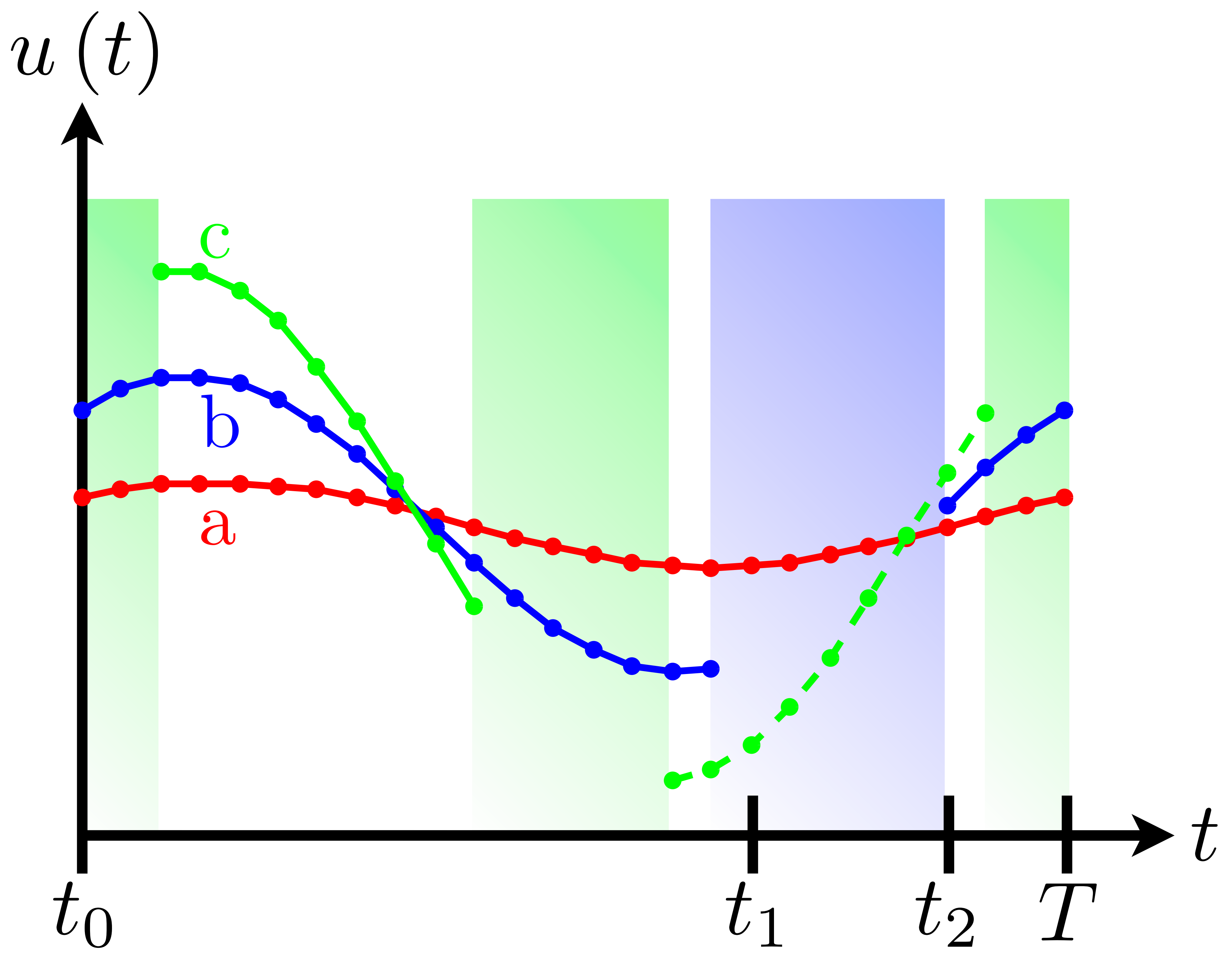
Three primary approaches:

1. Global expansion of the solution
2. Local expansion of the solution
3. Mixed approach

Restricted to uniform time samples

Oscillating Piston Example

- Node a never covered by piston
- Node b covered once per period
- Node c covered twice per period



∗ Leffell, J. I., Murman, S. M., and Pulliam, T. H. , "An Extension of the Time-Spectral Method to Overset Solvers," AIAA Paper 0637, Grapevine, Texas, January 2013
Leffell, J. I., "An Overset Time-Spectral Method for Relative Motion", PhD Thesis, Stanford University, June 2014

Hybrid Time-Spectral Method

Proposed Approach

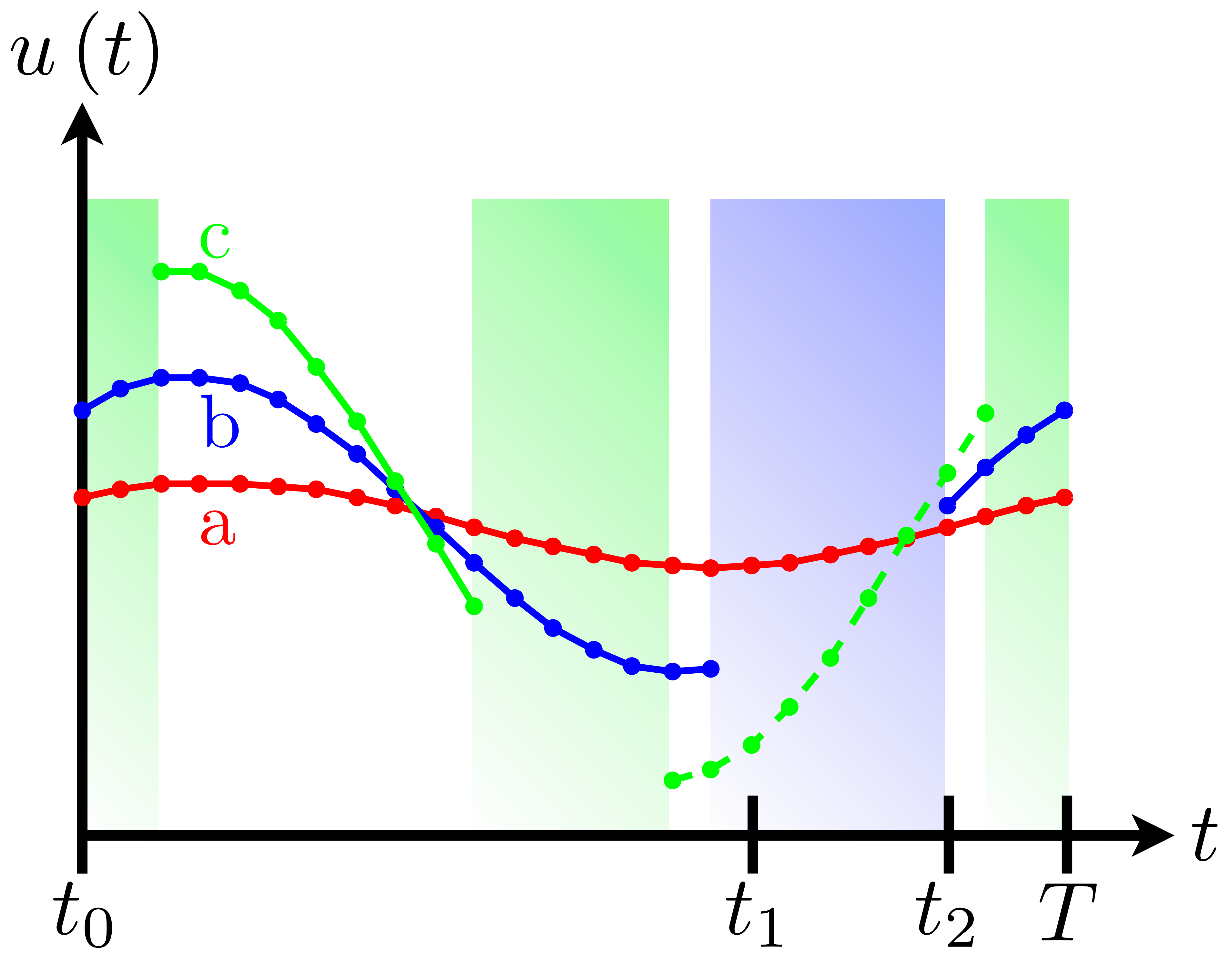
Three primary approaches:

1. Global expansion of the solution
2. Local expansion of the solution
3. Mixed approach

Restricted to uniform time samples

Oscillating Piston Example

- Node a never covered by piston
- Node b covered once per period
- Node c covered twice per period



∗ Leffell, J. I., Murman, S. M., and Pulliam, T. H. , "An Extension of the Time-Spectral Method to Overset Solvers," AIAA Paper 0637, Grapevine, Texas, January 2013
Leffell, J. I., "An Overset Time-Spectral Method for Relative Motion", PhD Thesis, Stanford University, June 2014

Hybrid Time-Spectral Method

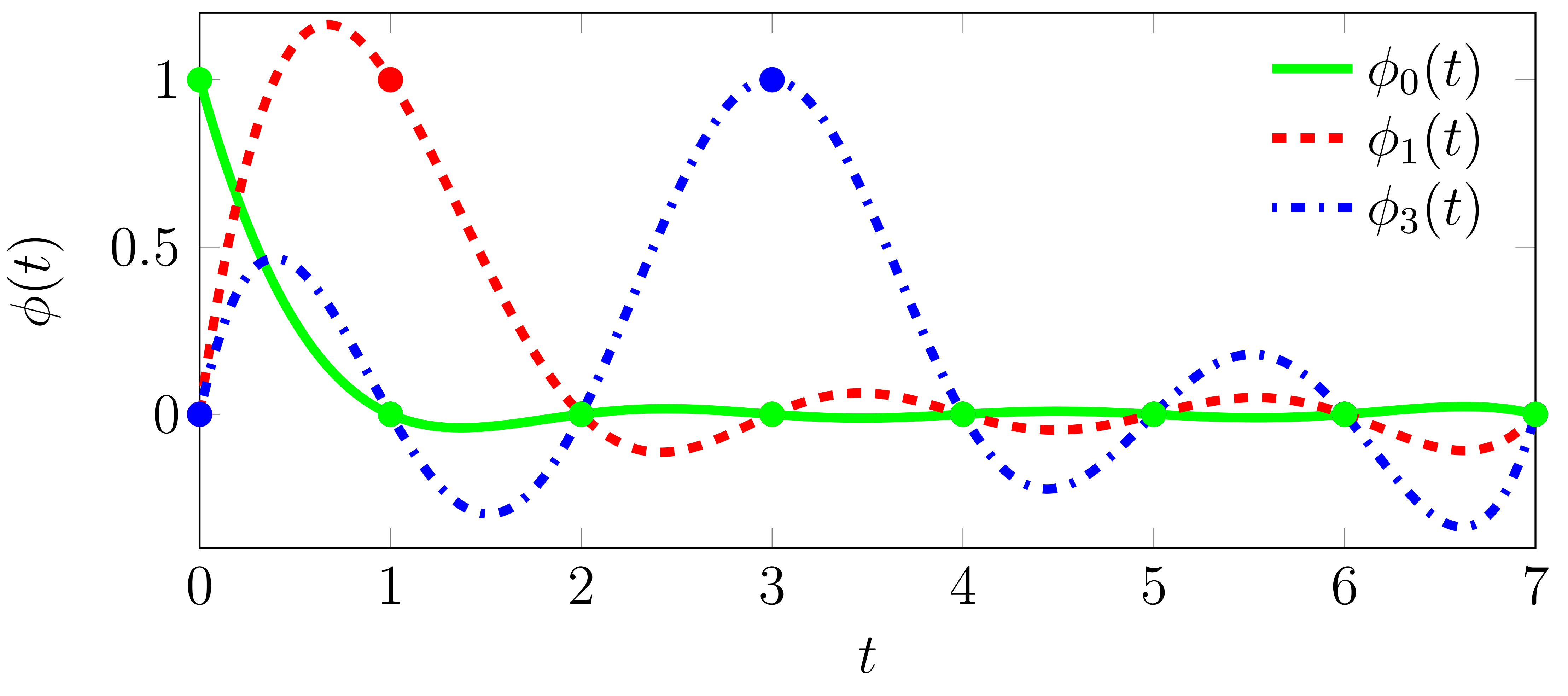
Local Expansion of the Solution - Bounded Interval

Barycentric Rational Interpolation

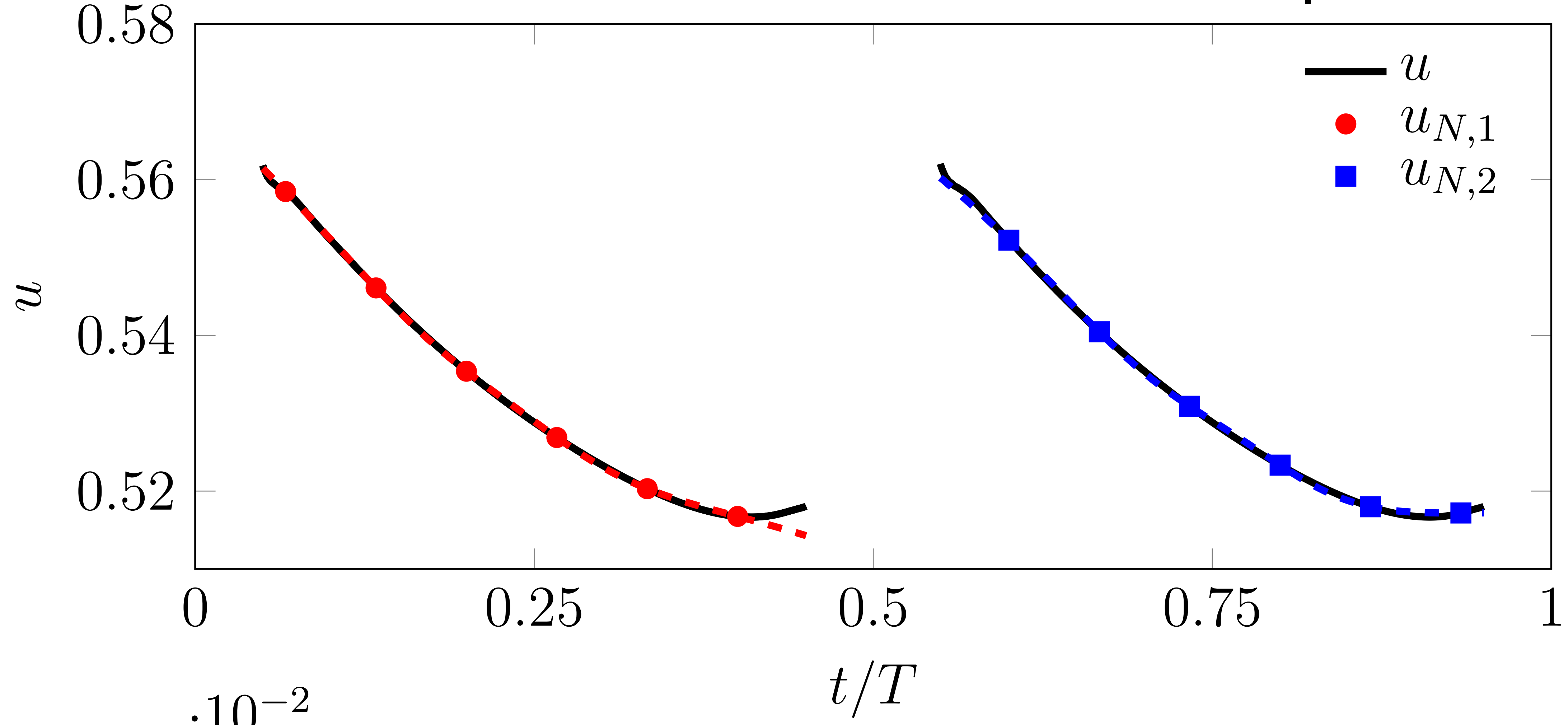
$$u_P(t) = \sum_{k=0}^{N} u_P(t_k)\,\phi_k(t)$$

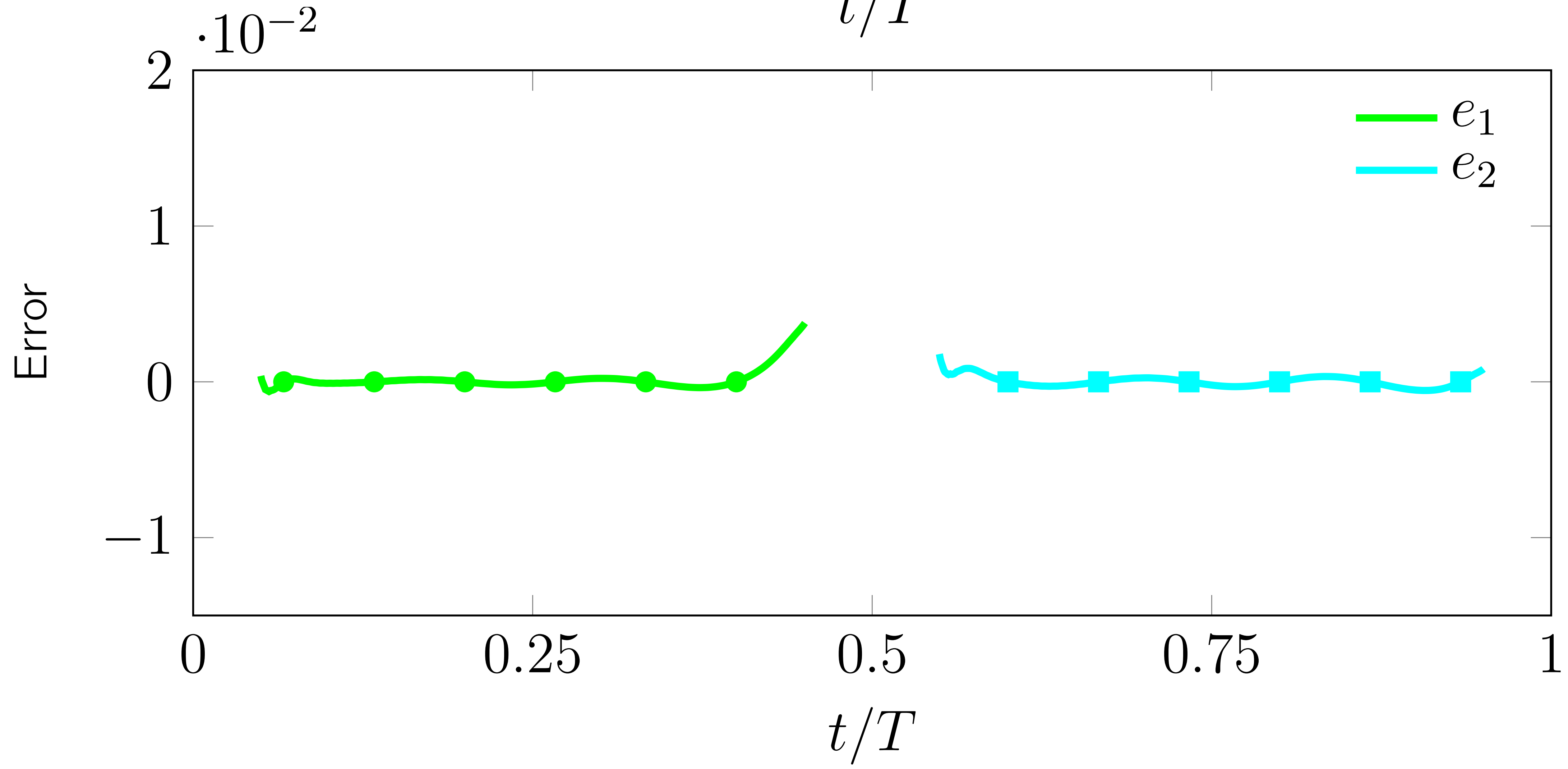
$$\phi_k(t) = \frac{\frac{w_k}{t-t_k}}{\sum_{j=0}^{N} \frac{w_j}{t-t_j}}$$

Sample basis functions



N = 15 Global Time Samples





∗ Leffell, J. I., Murman, S. M., and Pulliam, T. H. , "An Extension of the Time-Spectral Method to Overset Solvers," AIAA Paper 0637, Grapevine, Texas, January 2013
Leffell, J. I., "An Overset Time-Spectral Method for Relative Motion", PhD Thesis, Stanford University, June 2014

Hybrid Time-Spectral Method

Local Expansion of the Solution - Bounded Interval

Barycentric Rational Interpolation

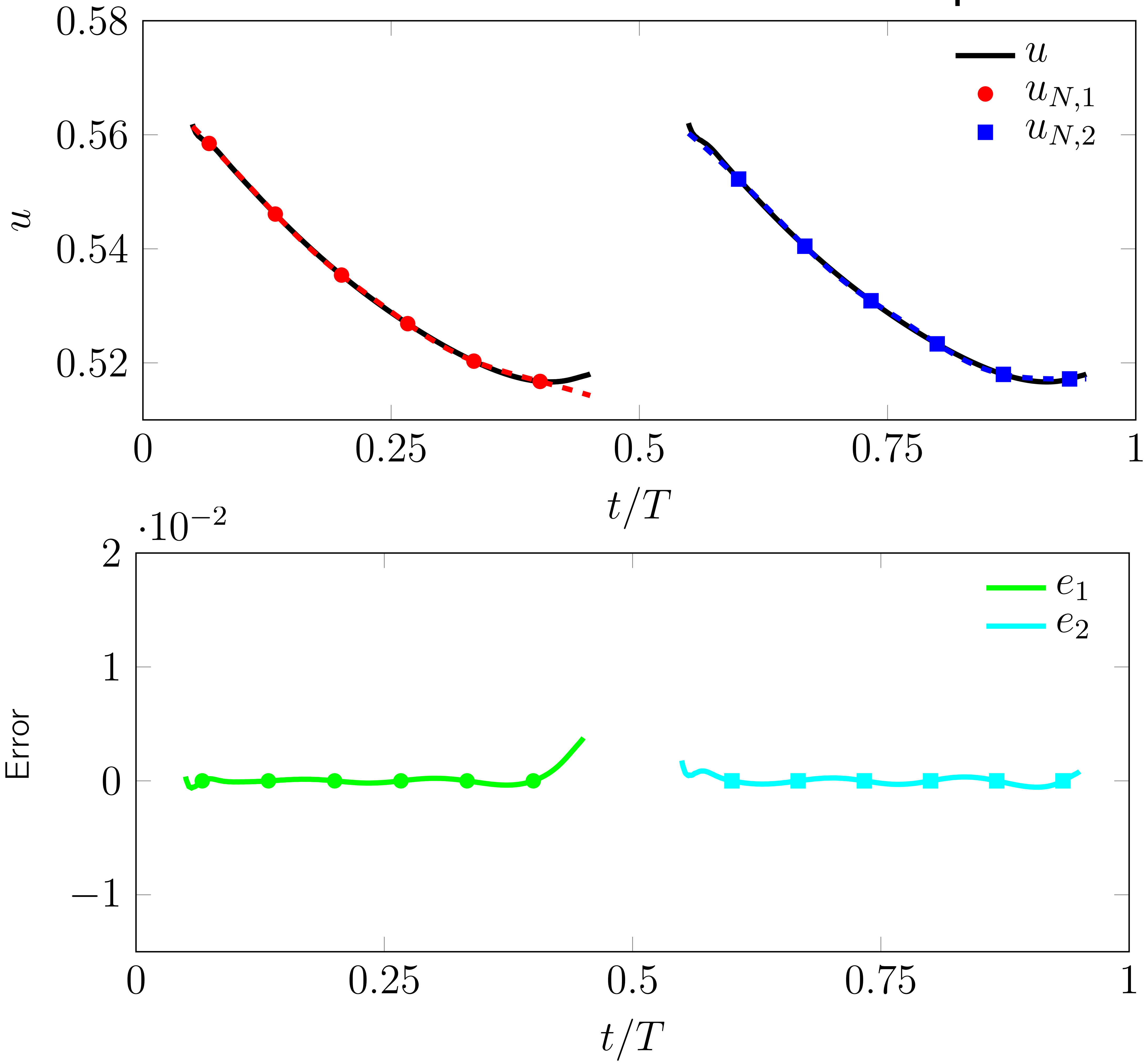
$$u_P\left(t\right) = \sum_{k=0}^{N} u_P\left(t_k\right)\phi_k\left(t\right)$$

$$\phi_k\left(t\right) = \frac{\frac{w_k}{t-t_k}}{\sum\limits_{j=0}^{N}\frac{w_j}{t-t_j}}$$

Analytic differentiation operator

$$\mathcal{D}_{jk} = \begin{cases} \frac{w_k}{w_j}\frac{1}{(t_j-t_k)} & \text{if } j \neq k \\ -\sum\limits_{i=0,i\neq k}^{N}\mathcal{D}_{ji} & \text{if } j = k \end{cases}$$

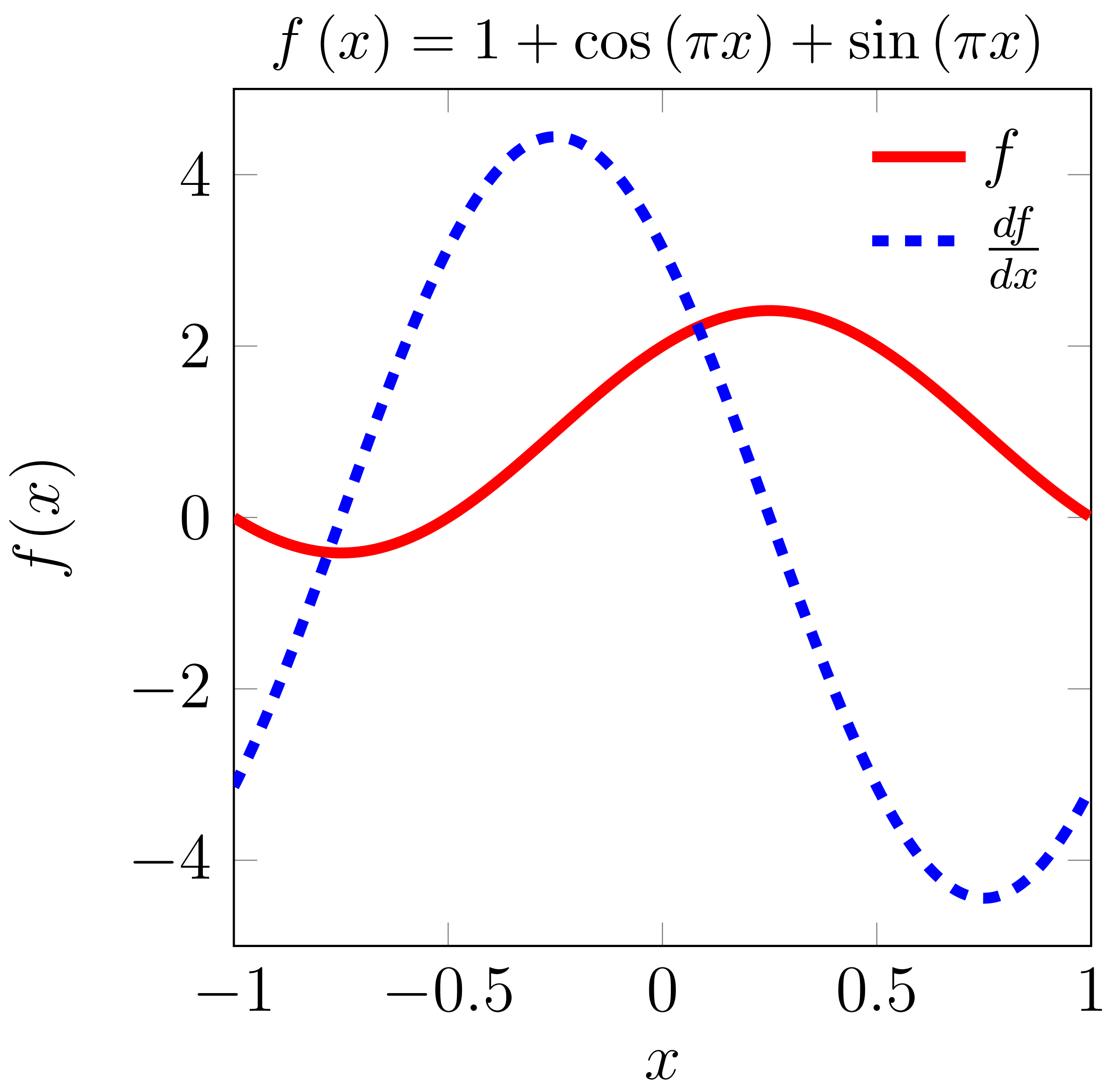
N = 15 Global Time Samples



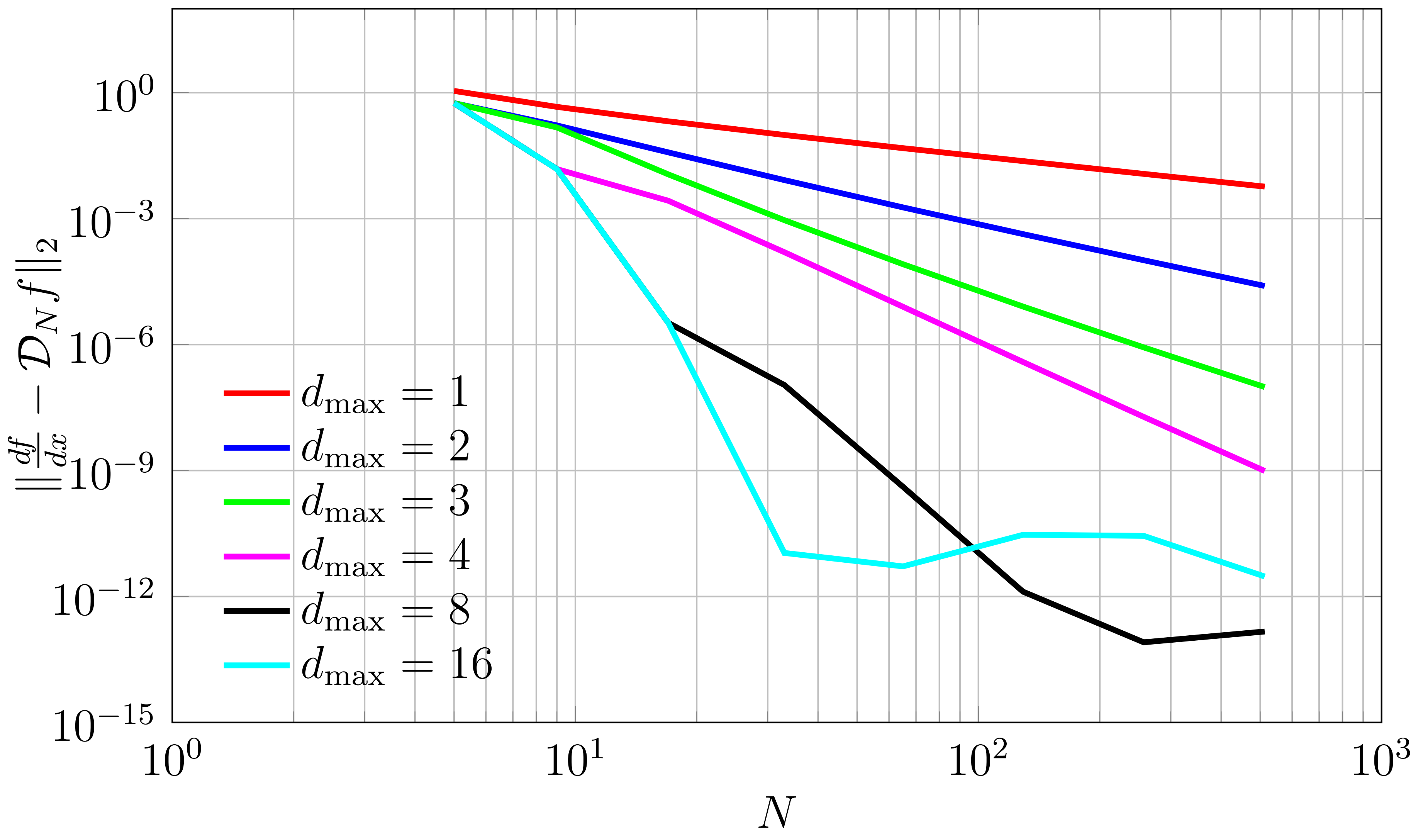
∗ Leffell, J. I., Murman, S. M., and Pulliam, T. H. , "An Extension of the Time-Spectral Method to Overset Solvers," AIAA Paper 0637, Grapevine, Texas, January 2013
Leffell, J. I., "An Overset Time-Spectral Method for Relative Motion", PhD Thesis, Stanford University, June 2014

Hybrid Time-Spectral Method

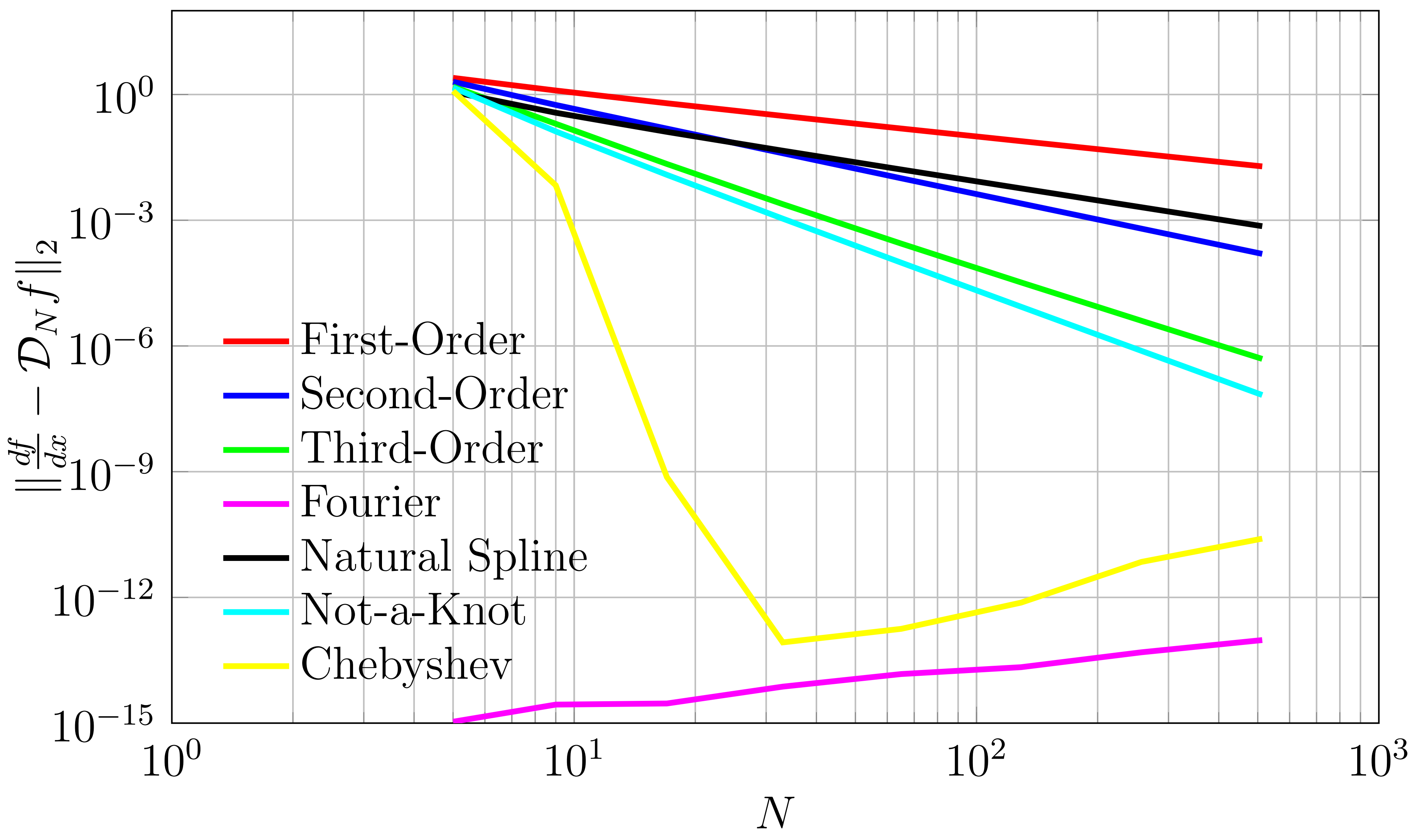
Differentiation Properties

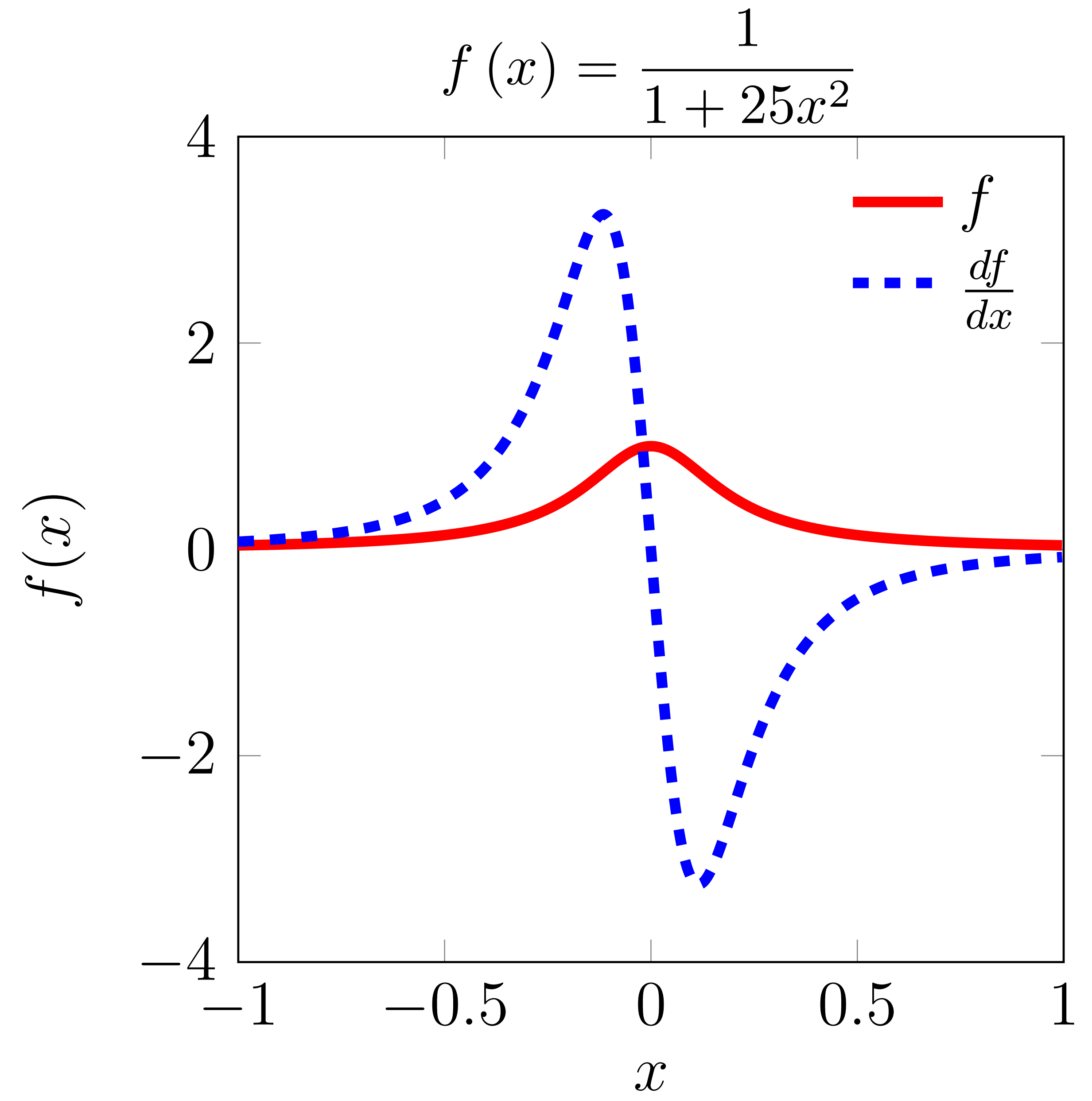


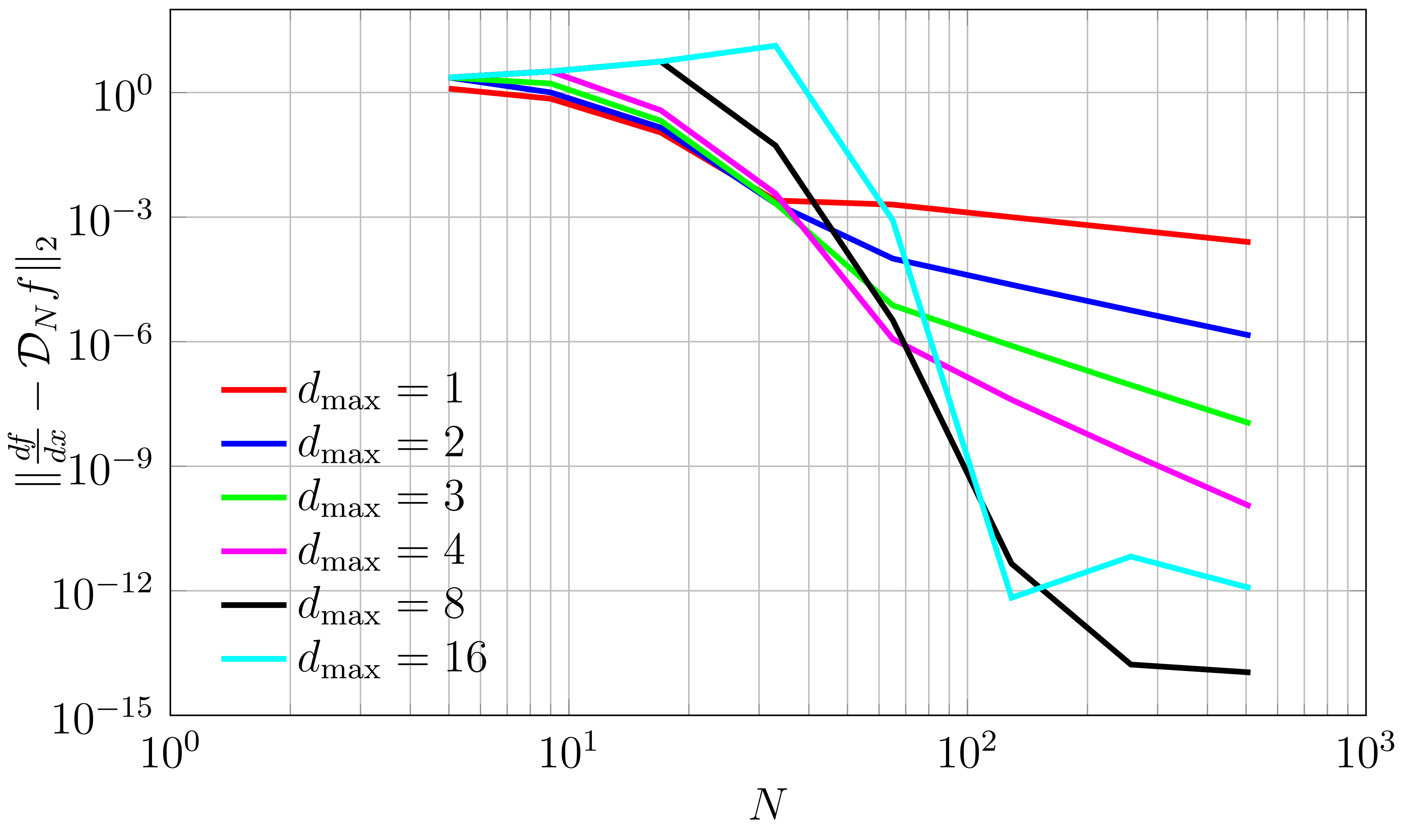
Barycentric Rational Interpolant

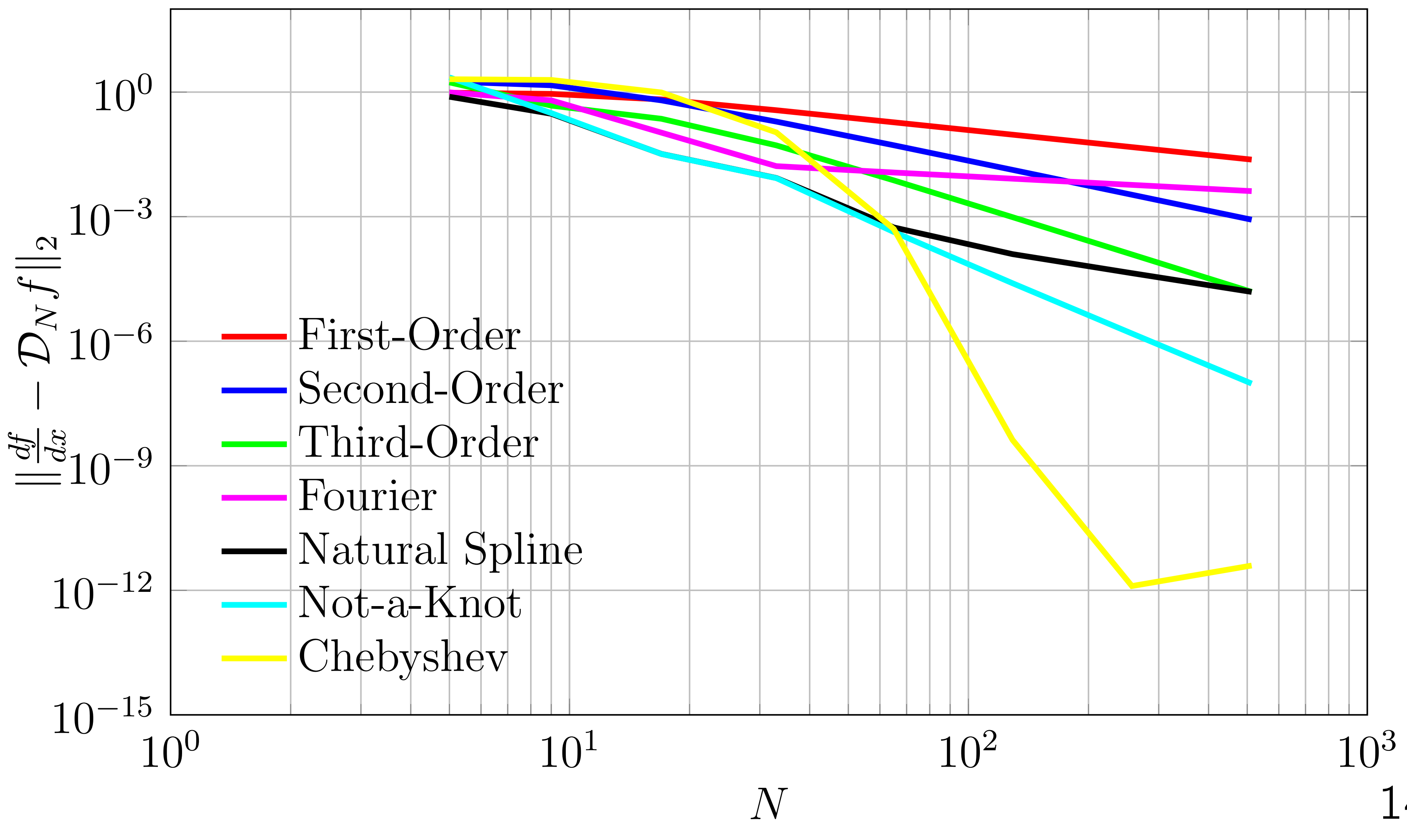


Other common operators









Outline

OVERFLOW

Augmented Time-Spectral Flow Solver

Time-derivative: finite-difference

$$\frac{Q^{s+1} - Q^{s}}{\Delta\tau} + \frac{3Q^{s+1} - 4Q^{n} + Q^{n-1}}{2\Delta t} + \left[\delta_x^L \mathcal{A} + \delta_y^L \mathcal{B}\right] \Delta Q = -\left[\delta_x^R F^s + \delta_y^R G^s\right]$$

Delta form, $\Delta Q = Q^{s+1} - Q^{s}$

$$\left[I + \Delta\tau \delta_x^L \mathcal{A} + \Delta\tau \delta_y^L \mathcal{B} + \Delta\tau \mathcal{D}_N\right] \Delta Q = -\Delta\tau \left[\delta_x^R F^s + \delta_y^R G^s + \mathcal{D}_N Q^s\right]$$

Approximate factorization

$$\left[I + \Delta\tau \delta_x^L \mathcal{A}\right] \left[I + \Delta\tau \delta_y^L \mathcal{B}\right] \left[I + \Delta\tau \mathcal{D}_N\right] \Delta Q = -\Delta\tau \left[\delta_x^R F^s + \delta_y^R G^s + \mathcal{D}_N Q^s\right] + \mathcal{O}\left(\Delta\tau^2\right)$$

$$\mathcal{L}_x \mathcal{L}_y \mathcal{L}_t \Delta Q = \mathcal{R}\left(Q^s\right)$$

N_{SD} + 1 direct inversions

$$\mathcal{L}_x \Delta\bar{Q} = \mathcal{R}\left(Q^s\right)$$

$$\mathcal{L}_y \Delta\bar{\bar{Q}} = \Delta\bar{Q}$$

$$\mathcal{L}_t \Delta Q = \Delta\bar{\bar{Q}}$$

OVERFLOW

Augmented Time-Spectral Flow Solver

Time-derivative: finite-difference $\longrightarrow$ spectrally-accurate global operator

$$\frac{Q^{s+1} - Q^s}{\Delta\tau} + \frac{3Q^{s+1} - 4Q^n + Q^{n-1}}{2\Delta t} + \left[\delta_x^L \mathcal{A} + \delta_y^L \mathcal{B}\right] \Delta Q = -\left[\delta_x^R F^s + \delta_y^R G^s\right]$$

$$\frac{Q^{s+1} - Q^s}{\Delta\tau} + \mathcal{D}_N Q^{s+1} + \left[\delta_x^L \mathcal{A} + \delta_y^L \mathcal{B}\right] \Delta Q = -\left[\delta_x^R F^s + \delta_y^R G^s\right]$$

Delta form, $\Delta Q = Q^{s+1} - Q^s$

$$\left[I + \Delta\tau\delta_x^L \mathcal{A} + \Delta\tau\delta_y^L \mathcal{B} + \Delta\tau\mathcal{D}_N\right] \Delta Q = -\Delta\tau \left[\delta_x^R F^s + \delta_y^R G^s + \mathcal{D}_N Q^s\right]$$

Approximate factorization

$$\left[I + \Delta\tau\delta_x^L \mathcal{A}\right] \left[I + \Delta\tau\delta_y^L \mathcal{B}\right] \left[I + \Delta\tau\mathcal{D}_N\right] \Delta Q = -\Delta\tau \left[\delta_x^R F^s + \delta_y^R G^s + \mathcal{D}_N Q^s\right] + \mathcal{O}\left(\Delta\tau^2\right)$$

$$\mathcal{L}_x \mathcal{L}_y \mathcal{L}_t \Delta Q = \mathcal{R}\left(Q^s\right)$$

N_{SD} + 1 direct inversions

$$\mathcal{L}_x \Delta\bar{Q} = \mathcal{R}\left(Q^s\right)$$

$$\mathcal{L}_y \Delta\bar{\bar{Q}} = \Delta\bar{Q}$$

$$\mathcal{L}_t \Delta Q = \Delta\bar{\bar{Q}}$$

OVERFLOW

Augmented Time-Spectral Flow Solver

Time-derivative: finite-difference ⟶ spectrally-accurate global operator

$$\frac{Q^{s+1}-Q^s}{\Delta\tau}+\frac{3Q^{s+1}-4Q^n+Q^{n-1}}{2\Delta t}+\left[\delta_x^L\mathcal{A}+\delta_y^L\mathcal{B}\right]\Delta Q=-\left[\delta_x^R F^s+\delta_y^R G^s\right]$$

$$\frac{Q^{s+1}-Q^s}{\Delta\tau}+\mathcal{D}_N Q^{s+1}+\left[\delta_x^L\mathcal{A}+\delta_y^L\mathcal{B}\right]\Delta Q=-\left[\delta_x^R F^s+\delta_y^R G^s\right]$$

Delta form, $\Delta Q = Q^{s+1}-Q^s$

$$\left[I+\Delta\tau\delta_x^L\mathcal{A}+\Delta\tau\delta_y^L\mathcal{B}+\Delta\tau\mathcal{D}_N\right]\Delta Q=-\Delta\tau\left[\delta_x^R F^s+\delta_y^R G^s+\mathcal{D}_N Q^s\right]$$

Approximate factorization

$$\left[I+\Delta\tau\delta_x^L\mathcal{A}\right]\left[I+\Delta\tau\delta_y^L\mathcal{B}\right]\left[I+\Delta\tau\mathcal{D}_N\right]\Delta Q=-\Delta\tau\left[\delta_x^R F^s+\delta_y^R G^s+\mathcal{D}_N Q^s\right]+\mathcal{O}\left(\Delta\tau^2\right)$$

$$\mathcal{L}_x\mathcal{L}_y\mathcal{L}_t\Delta Q=\mathcal{R}\left(Q^s\right)$$

N_{SD} + 1 direct inversions

$$\mathcal{L}_x\Delta\bar{Q}=\mathcal{R}\left(Q^s\right)$$

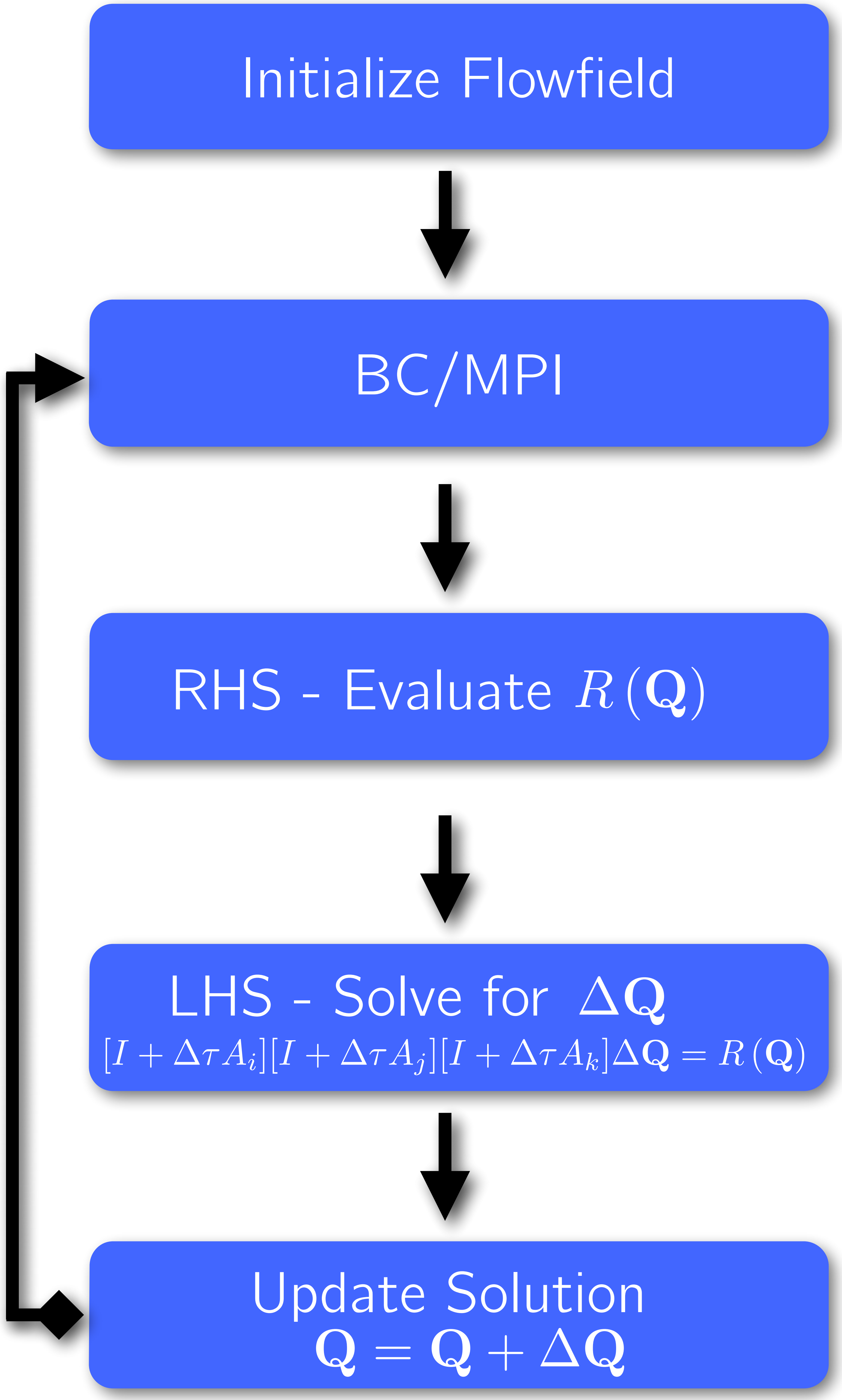
$$\mathcal{L}_y\Delta\bar{\bar{Q}}=\Delta\bar{Q}$$

$$\mathcal{L}_t\Delta Q=\Delta\bar{\bar{Q}}$$

$$\begin{aligned}\tilde{\mathcal{R}} &\leftarrow FFT\left(\Delta\bar{\bar{Q}}\right)\\ \Delta\tilde{Q}_k &\leftarrow \frac{\tilde{\mathcal{R}}}{1+\Delta\tau i\omega k}, \quad \forall k\\ \Delta Q &\leftarrow IFFT\left(\Delta\tilde{Q}\right)\end{aligned}$$

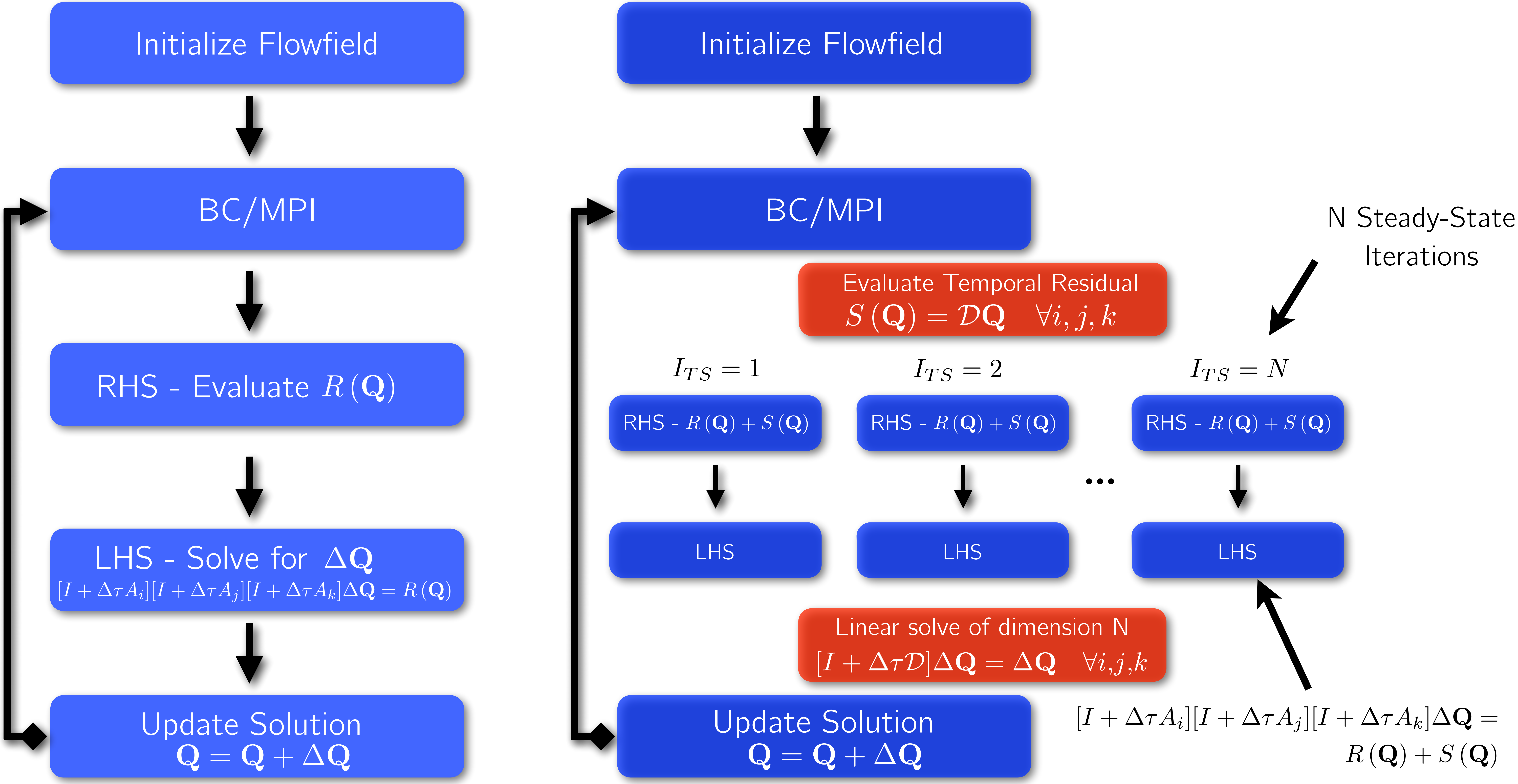
OVERFLOW

High-level Comparison



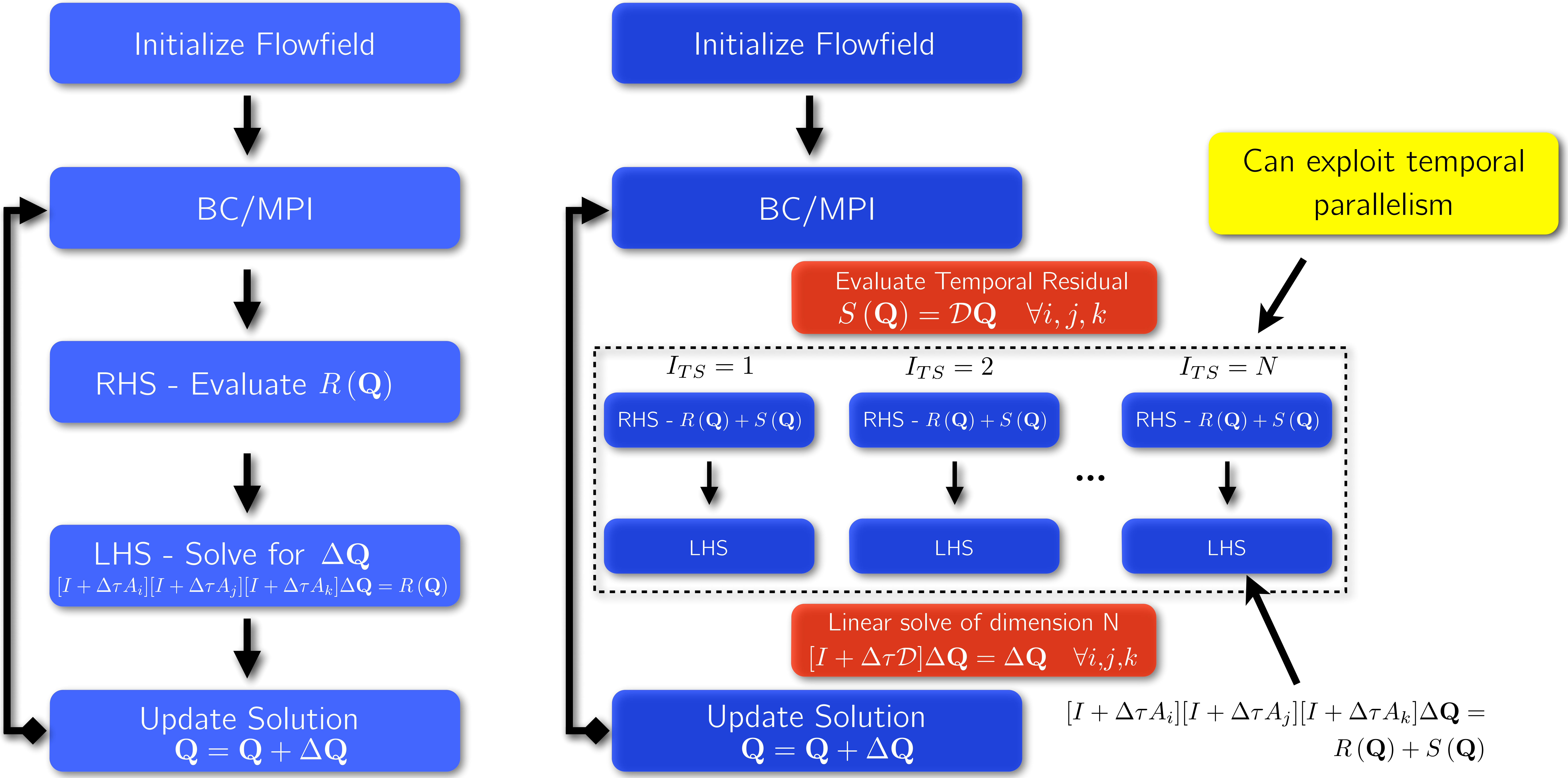
OVERFLOW

High-level Comparison



OVERFLOW

High-level Comparison

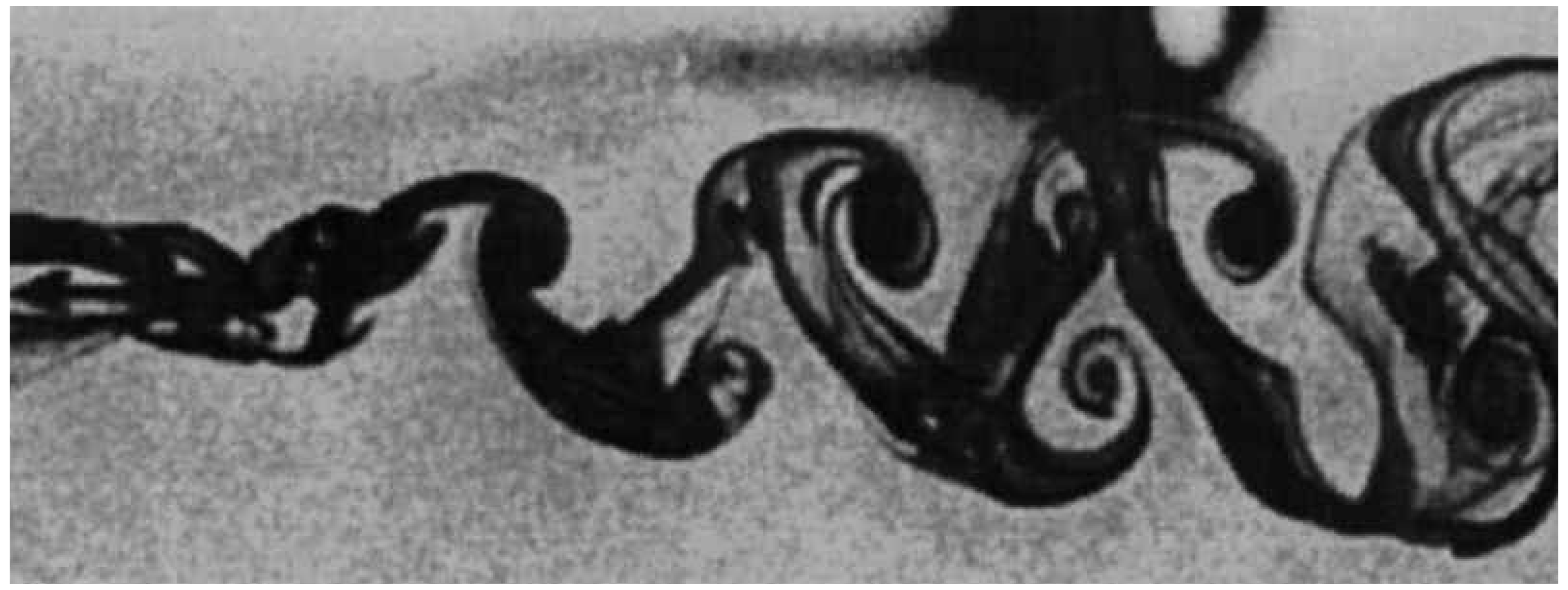


Outline

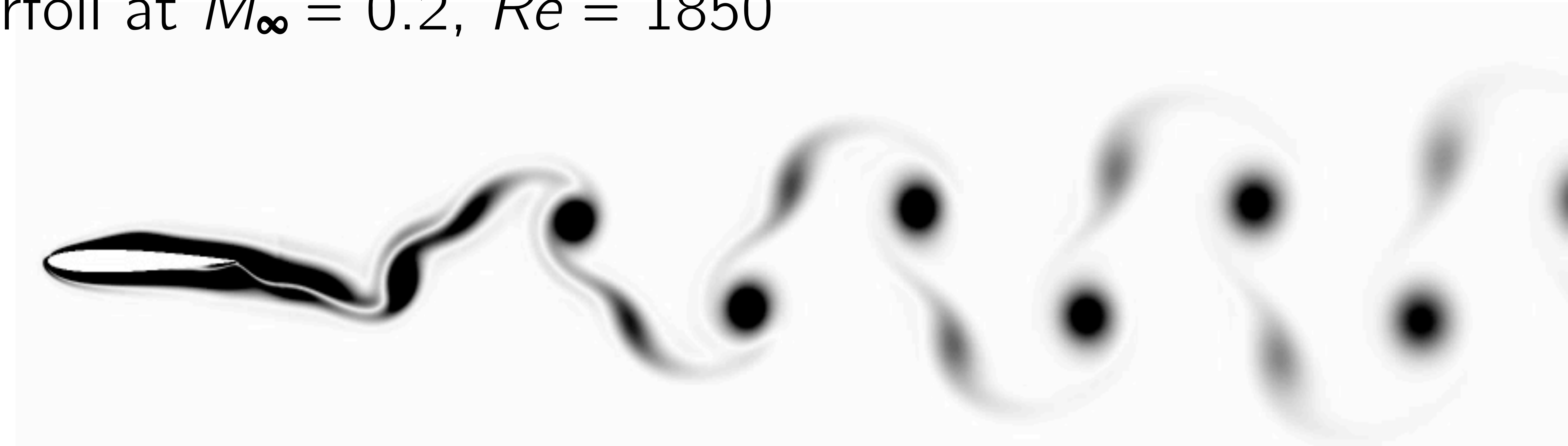
Two-Dimensional Oscillating Airfoils

Laminar Plunging NACA 0012 airfoil at $M_\infty = 0.2$, $Re = 1850$

Experimental



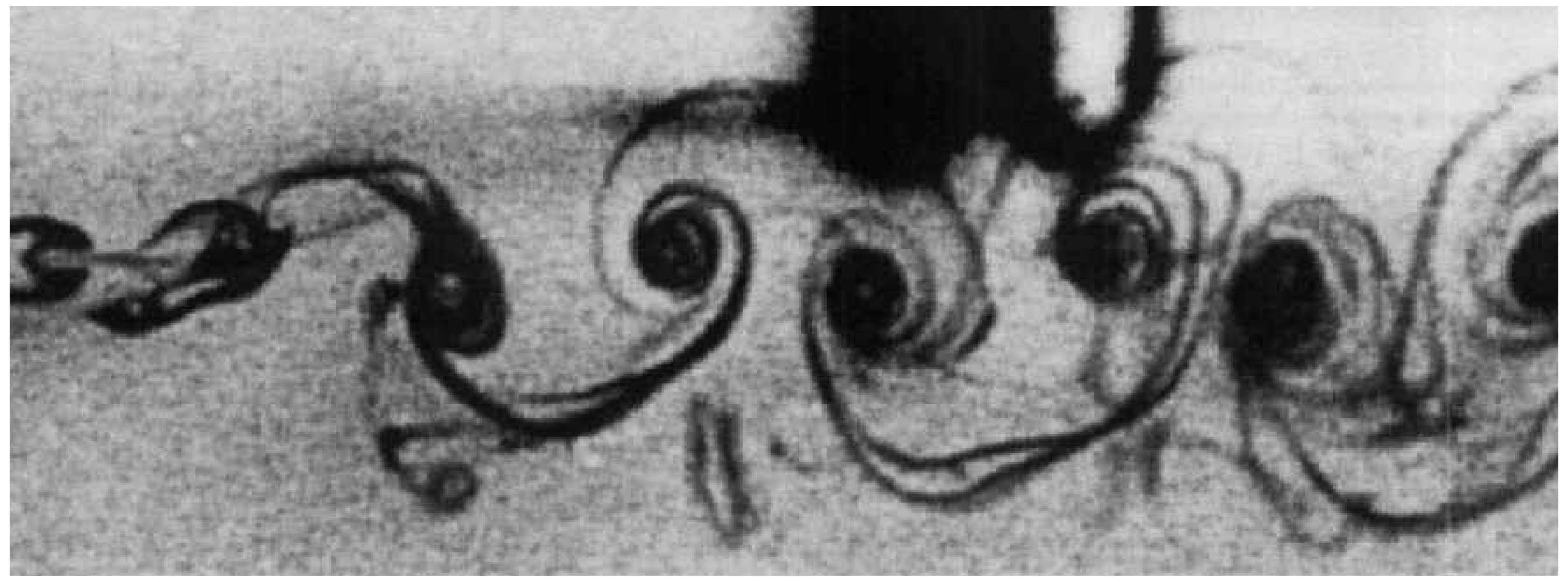
Jones et al. ['98]



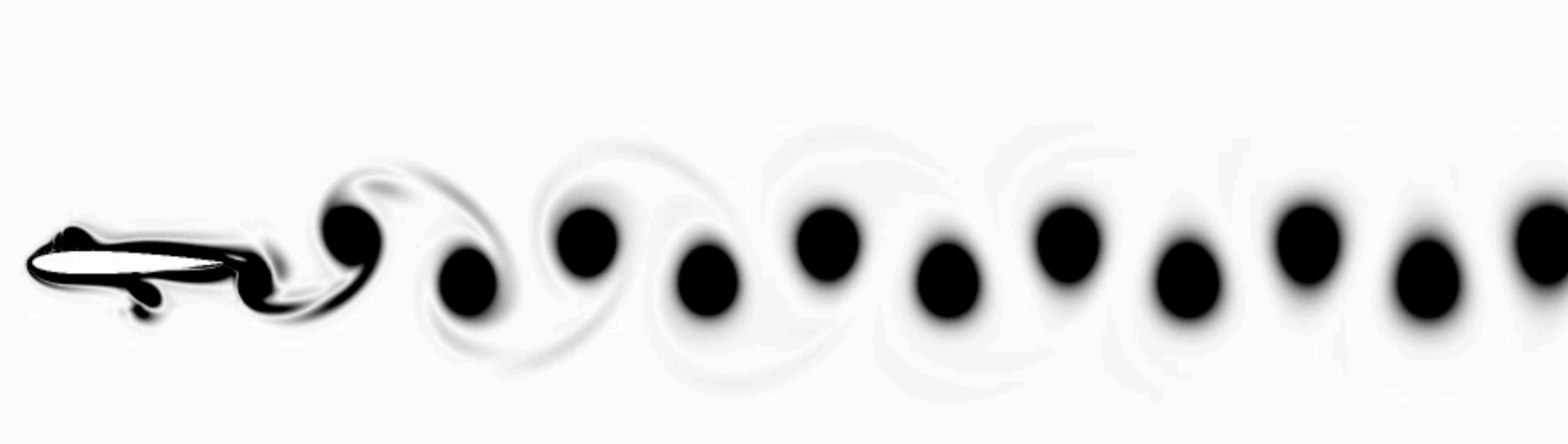
Drag Producing Case, $St = 0.288$

$k = 3.6$, $h = 0.08$

Experimental



Jones et al. ['98]



Thrust Producing Case, $St = 0.6$

$k = 6.0$, $h = 0.1$

Two-Dimensional Oscillating Airfoils

Laminar Plunging NACA 0012 airfoil at $M_{\infty} = 0.2$, $Re = 1850$
Drag Producing Case, $St = 0.288$, $k = 3.6$, $h = 0.08$

Rigid Motion

Modes/Samples

K = 1, N = 3

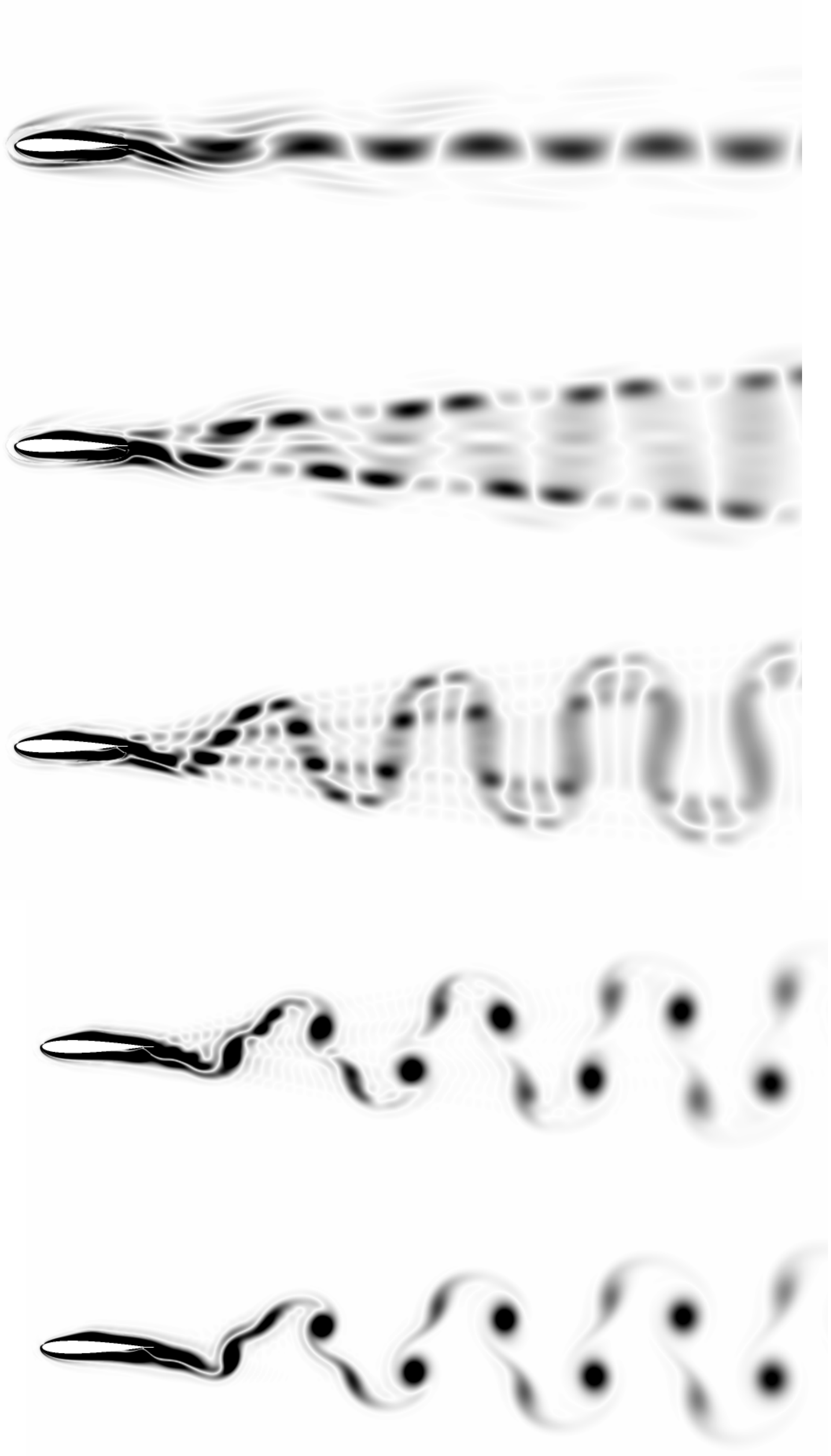
K = 2, N = 5

K = 4, N = 9

K = 8, N = 17

K = 16, N = 33

Relative Motion

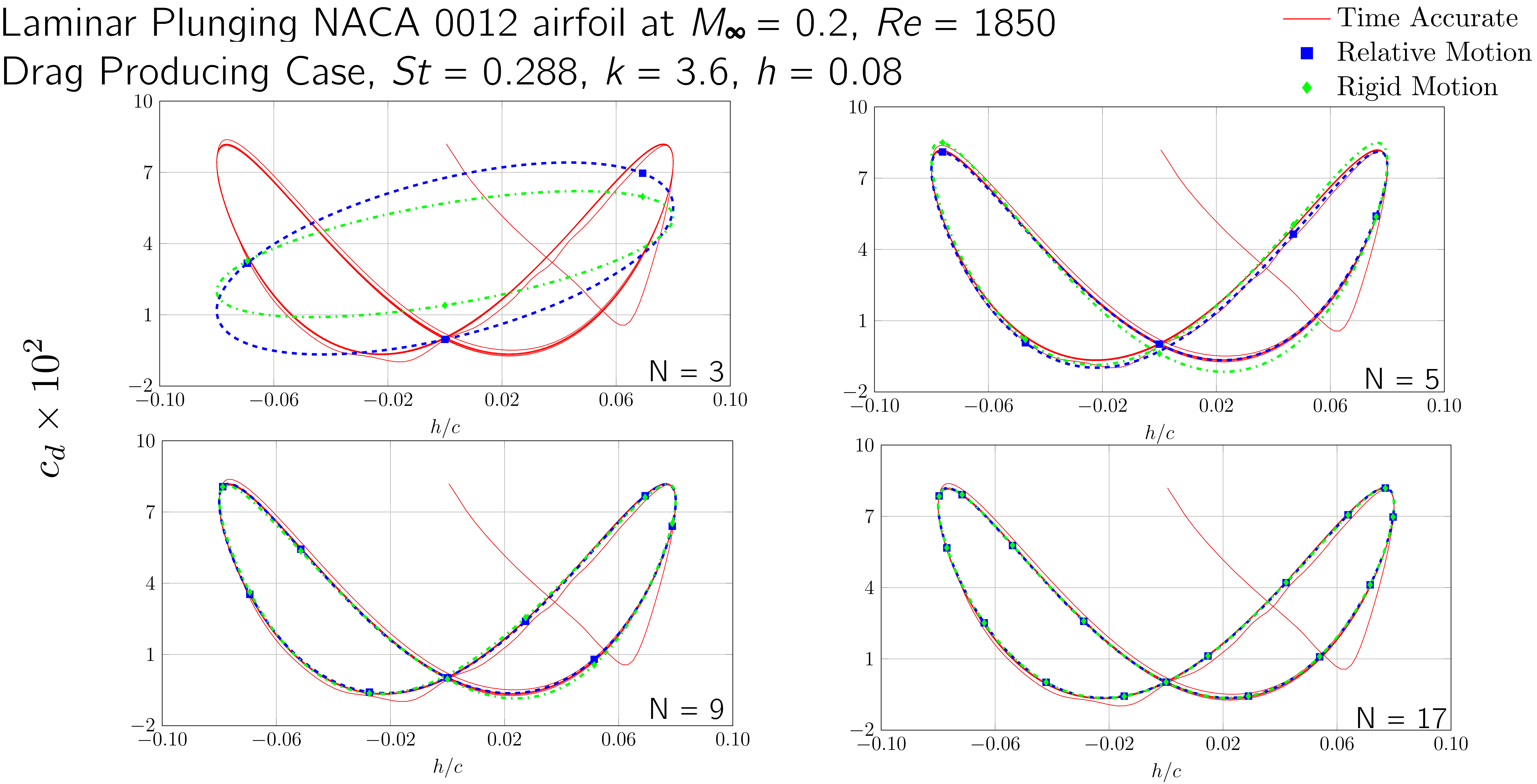


No Spectral Vanishing Viscosity Required for either Rigid or Relative Motion

Two-Dimensional Oscillating Airfoils

Laminar Plunging NACA 0012 airfoil at $M_\infty = 0.2$, $Re = 1850$

Drag Producing Case, $St = 0.288$, $k = 3.6$, $h = 0.08$

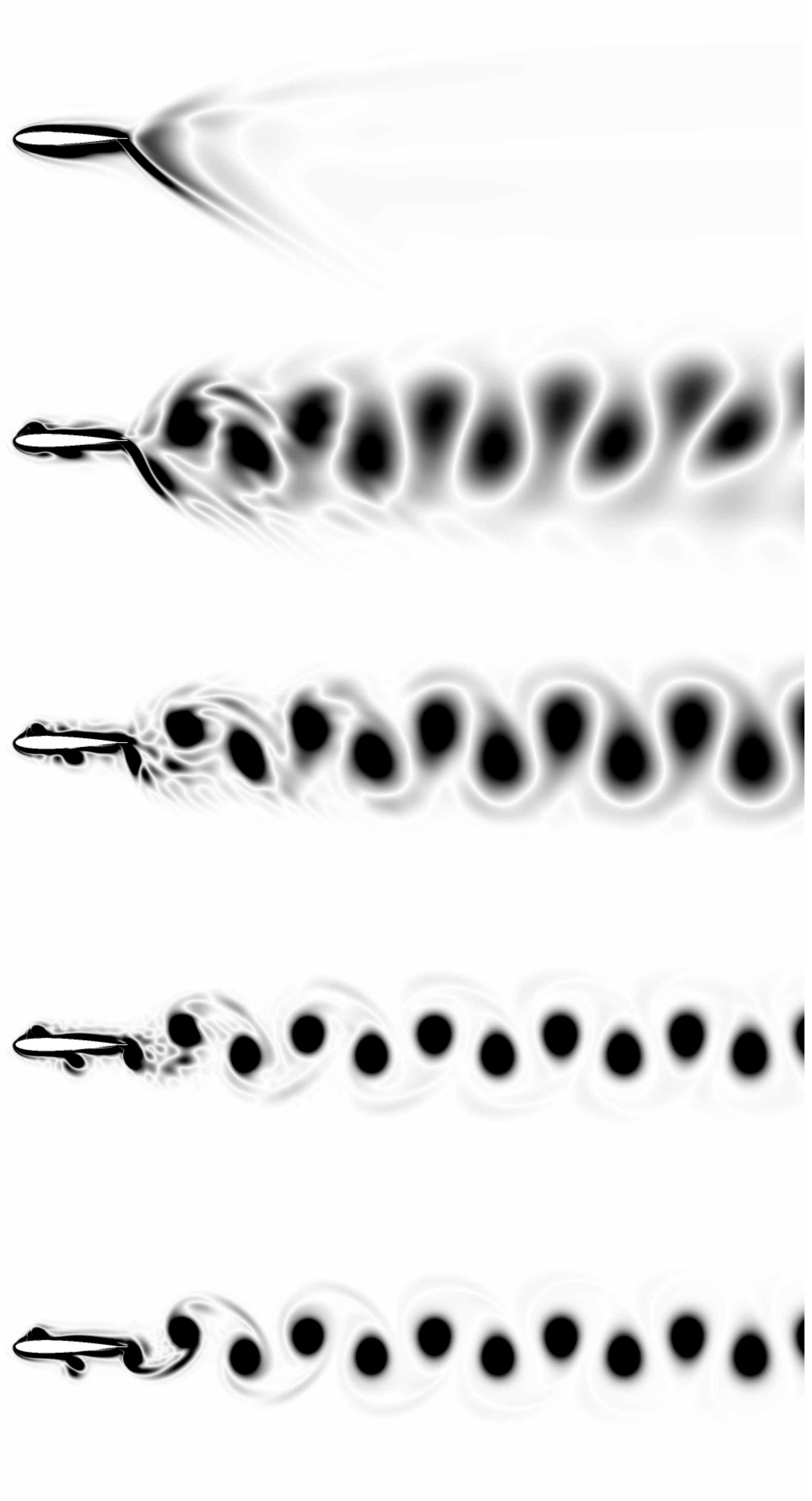
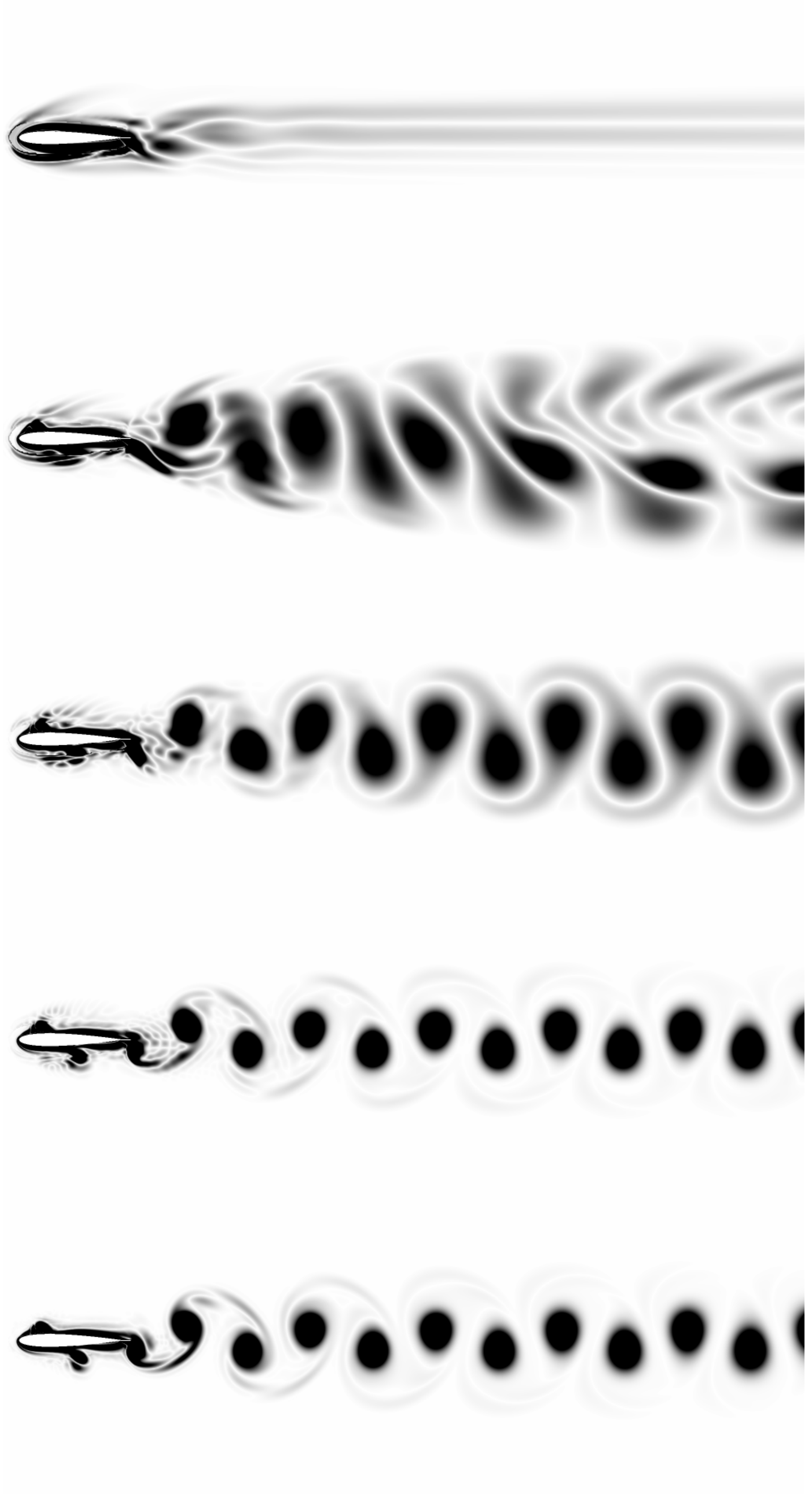


No Spectral Vanishing Viscosity Required for either Rigid or Relative Motion

Two-Dimensional Oscillating Airfoils

Laminar Plunging NACA 0012 airfoil at $M_\infty = 0.2$, $Re = 1850$

Thrust Producing Case, $St = 0.6$, $k = 6.0$, $h = 0.1$

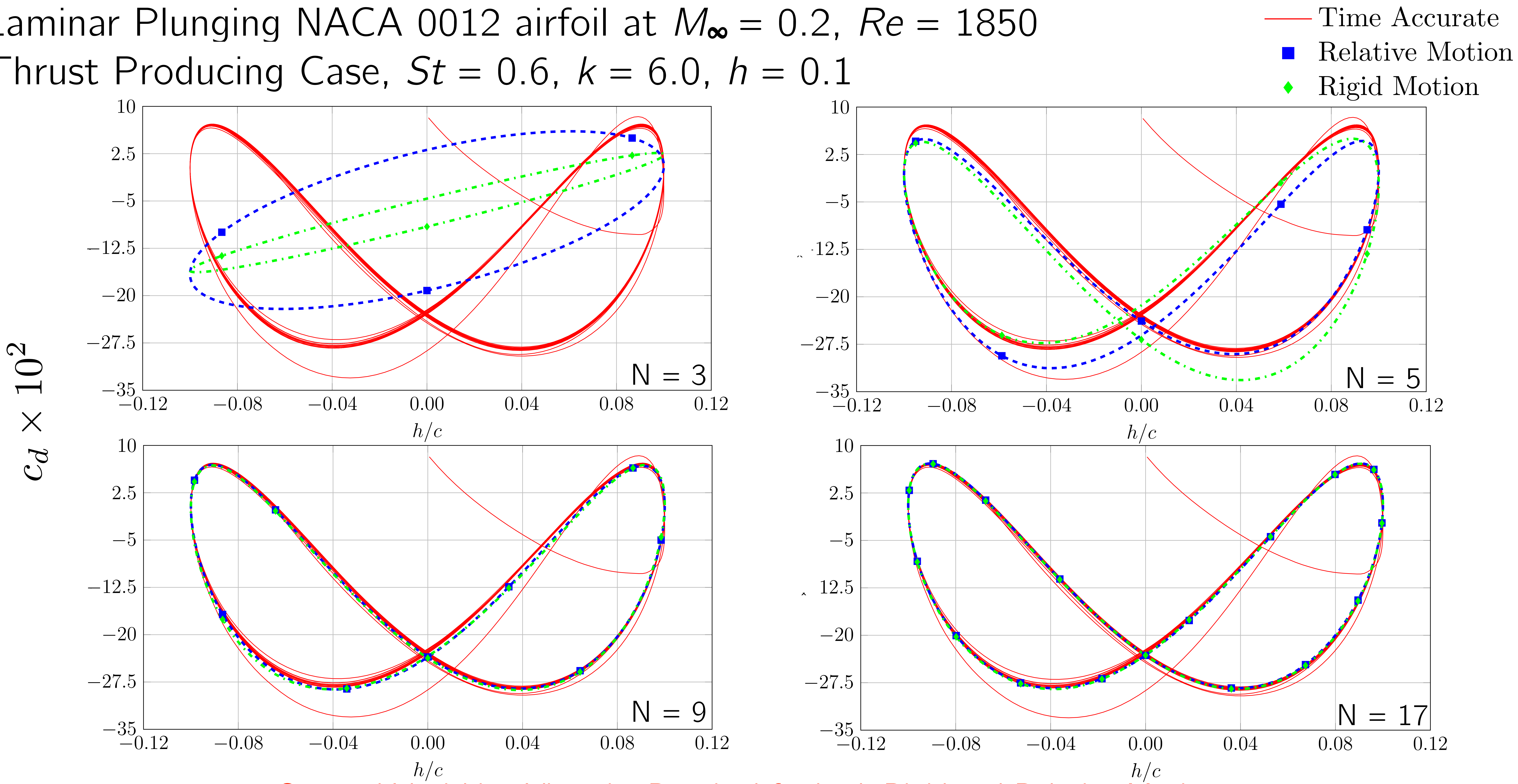
Rigid Motion	Modes/Samples	Relative Motion
	K = 1, N = 3 K = 2, N = 5 K = 4, N = 9 K = 8, N = 17 K = 16, N = 33	

Spectral Vanishing Viscosity Required for both Rigid and Relative Motion

Two-Dimensional Oscillating Airfoils

Laminar Plunging NACA 0012 airfoil at $M_\infty = 0.2$, $Re = 1850$

Thrust Producing Case, $St = 0.6$, $k = 6.0$, $h = 0.1$



Spectral Vanishing Viscosity Required for both Rigid and Relative Motion

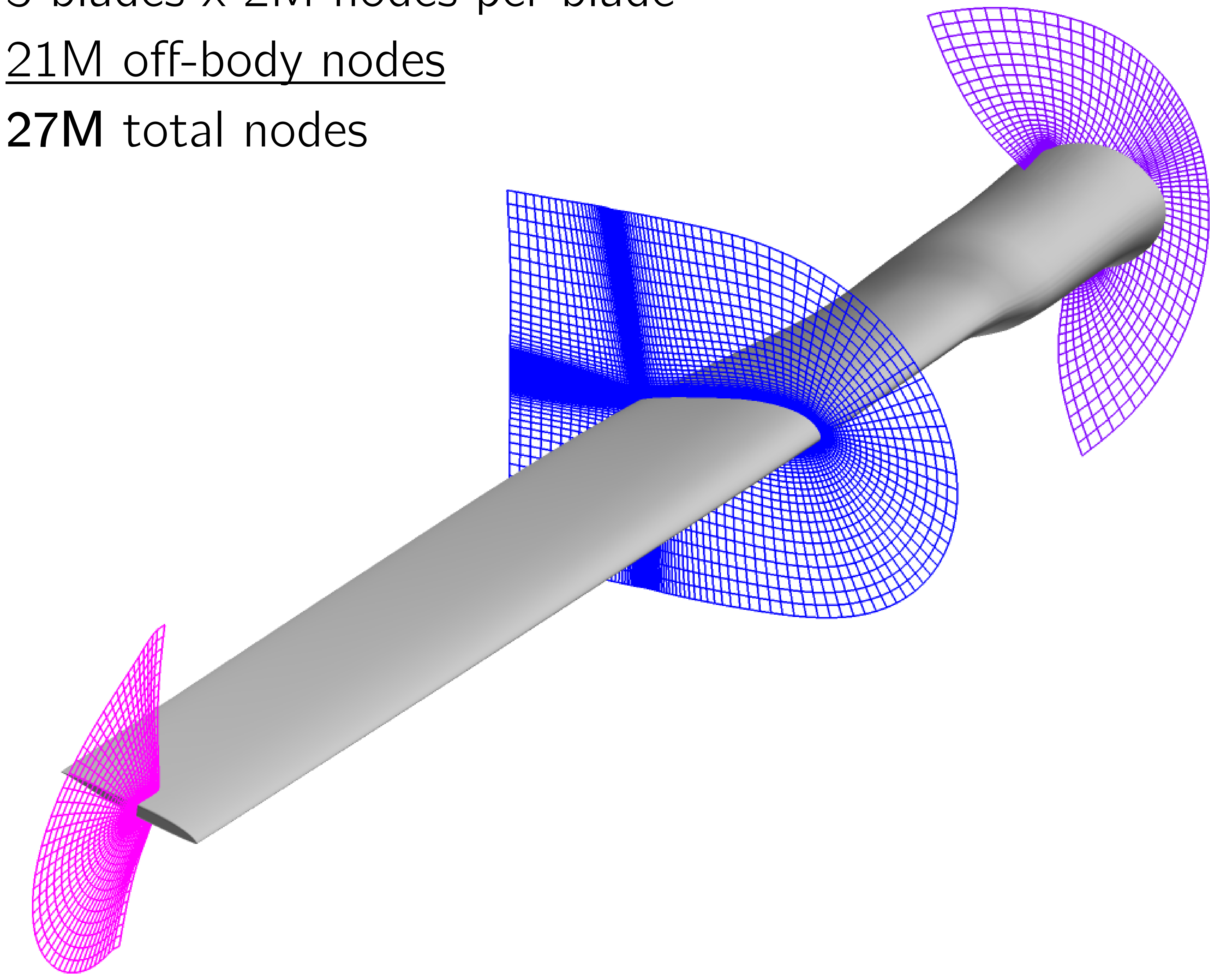
Three-Dimensional V22 TRAM

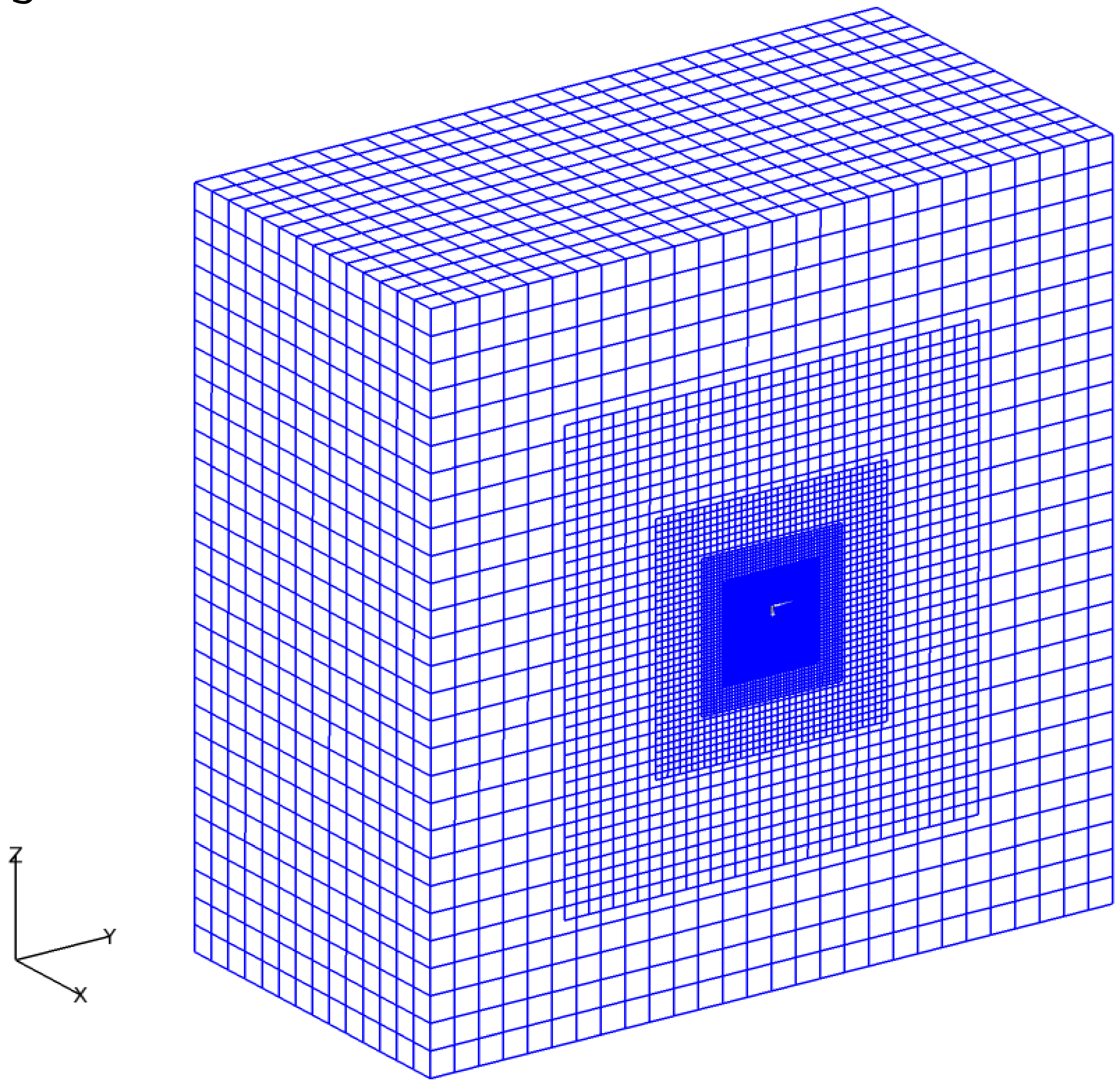
Hover - $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$, 14 degree collective

3 blades x 2M nodes per blade

<u>21M off-body nodes</u>

27M total nodes





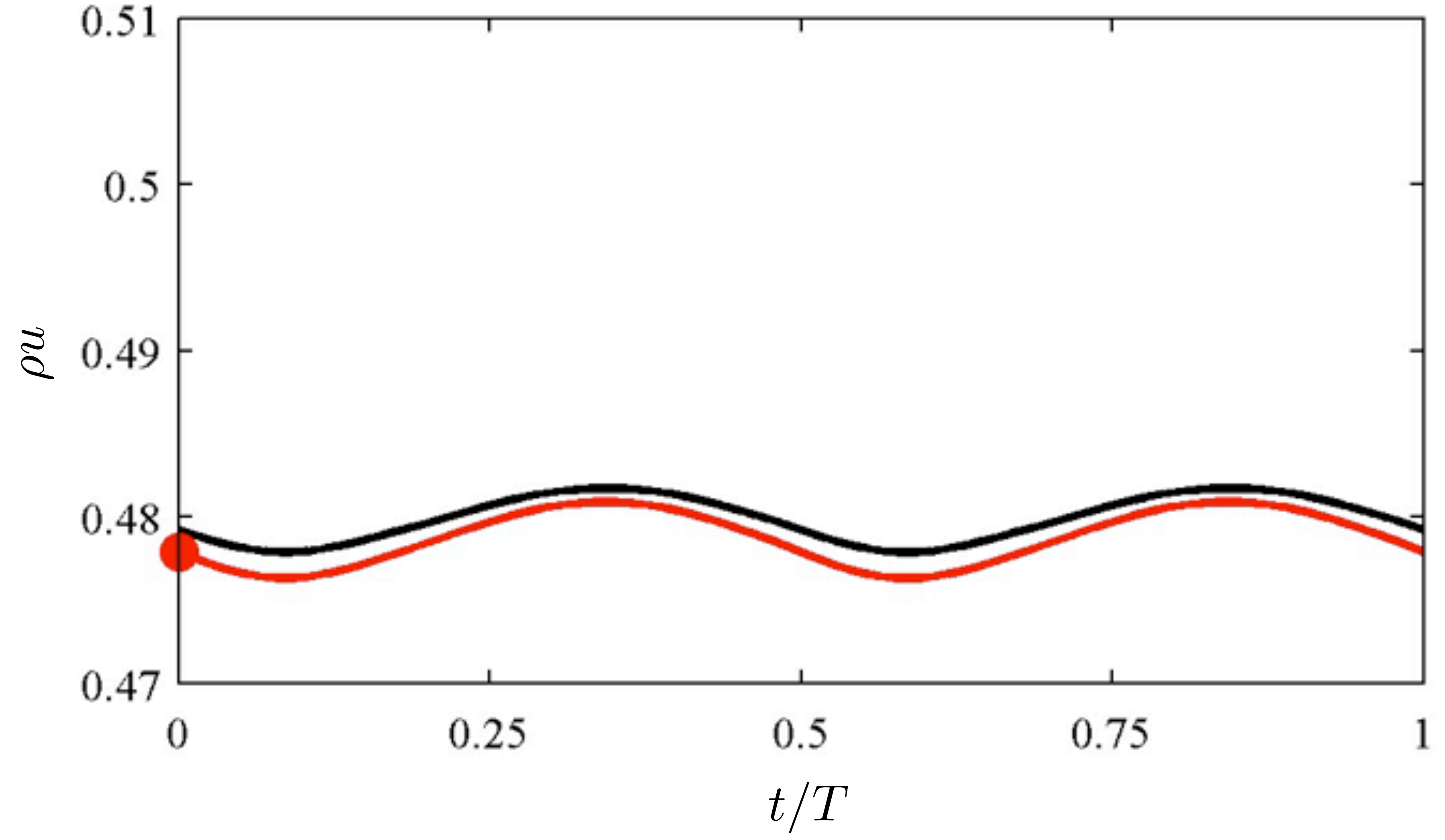
All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Two-Dimensional Oscillating Airfoils

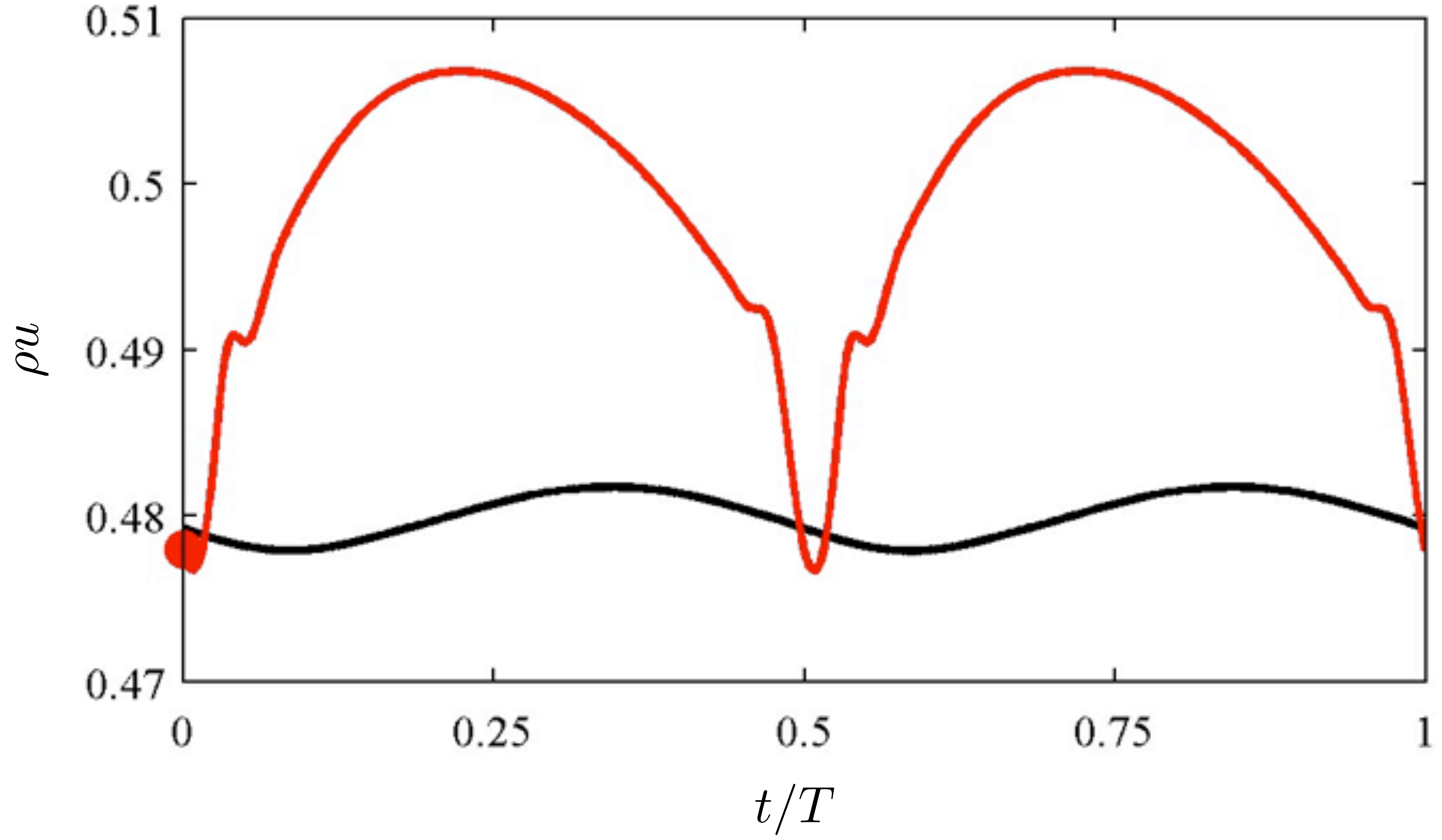
Inviscid Plunging NACA 0012 airfoil at $M_\infty = 0.5$

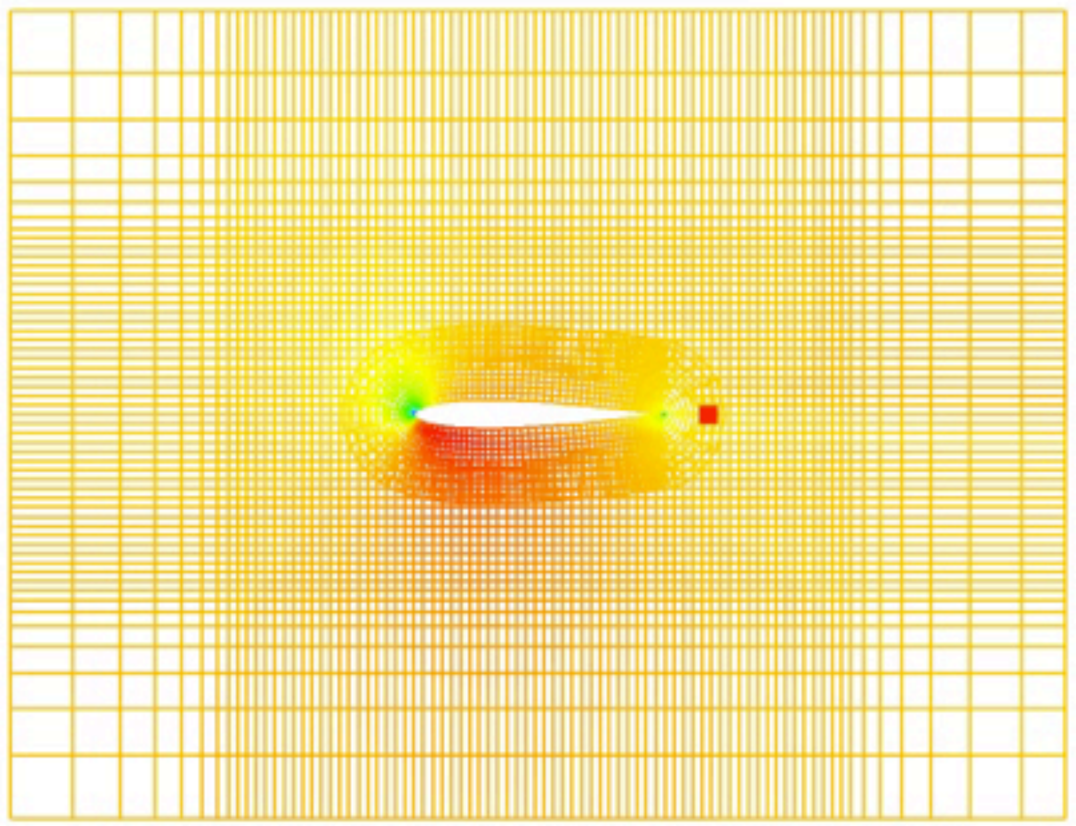
Near-Body Grid
Off-Body Grid

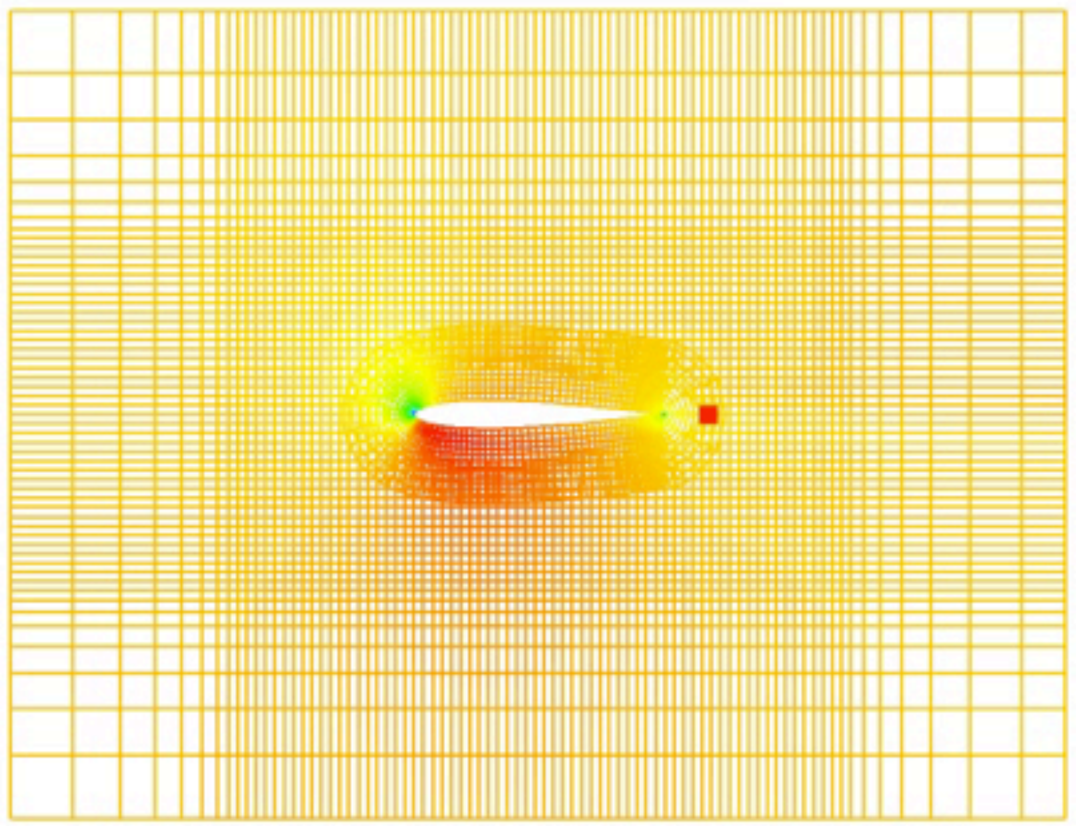
Rigid Motion



Relative Motion

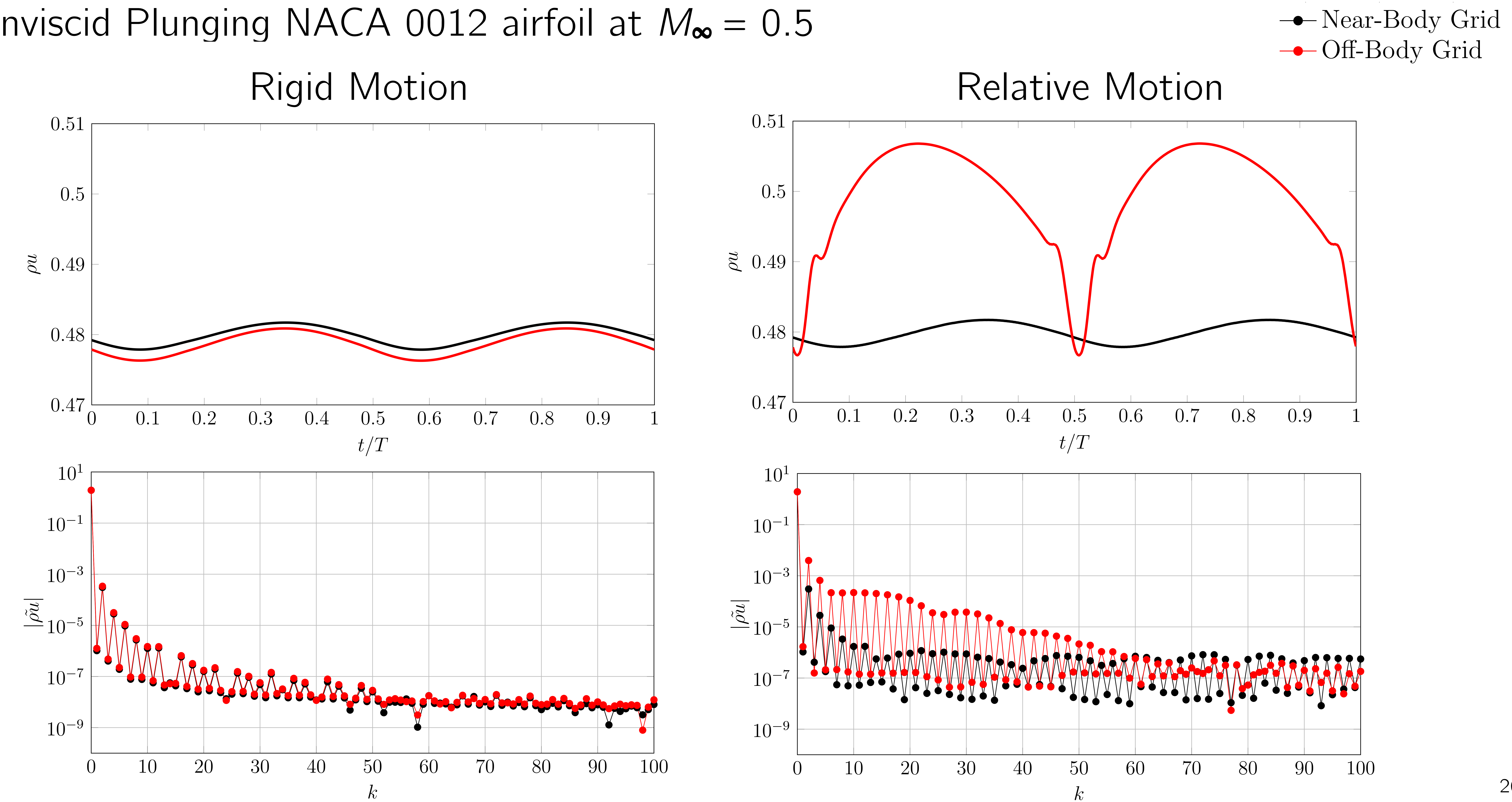






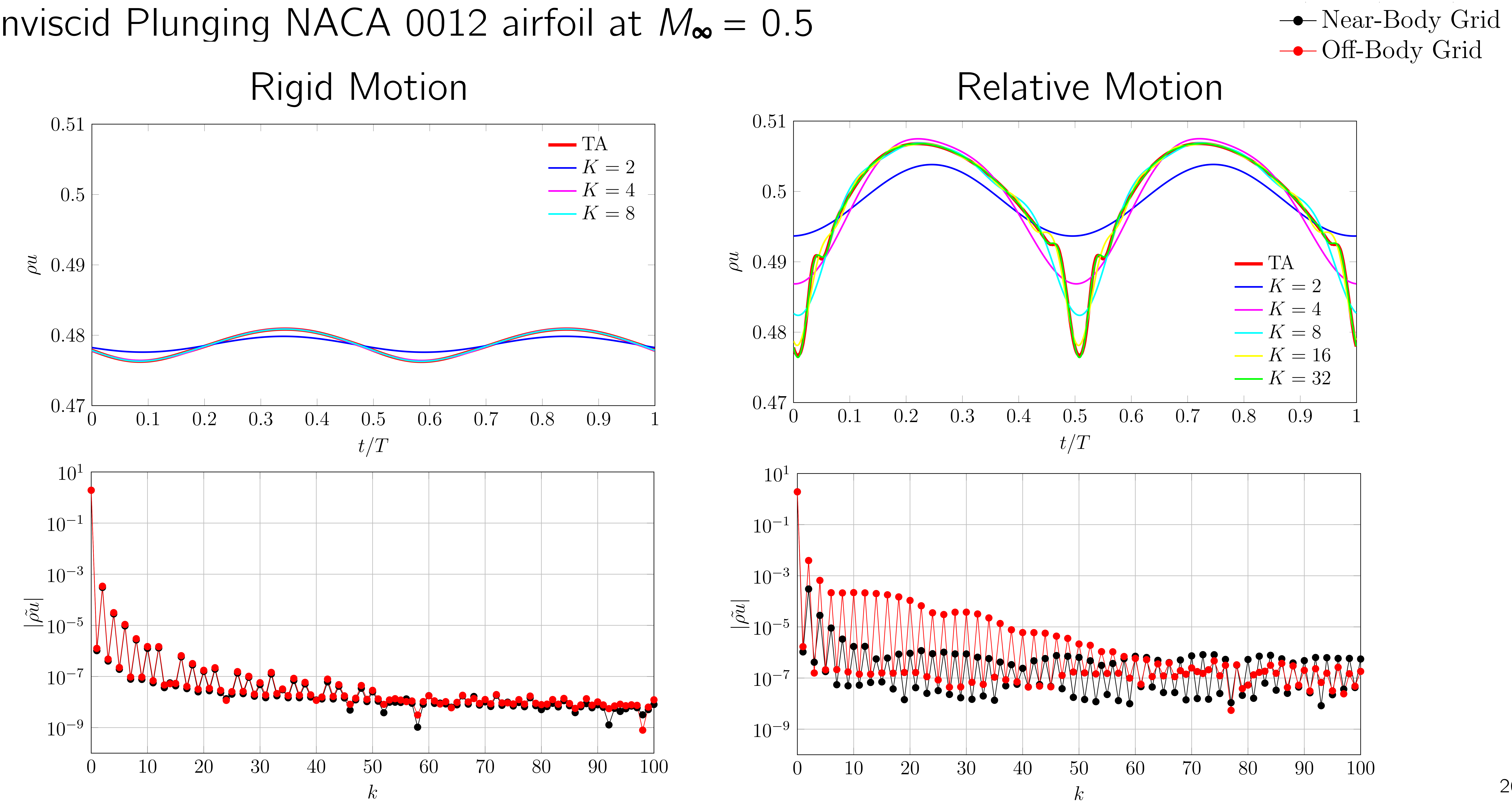
Two-Dimensional Oscillating Airfoils

Inviscid Plunging NACA 0012 airfoil at $M_\infty = 0.5$



Two-Dimensional Oscillating Airfoils

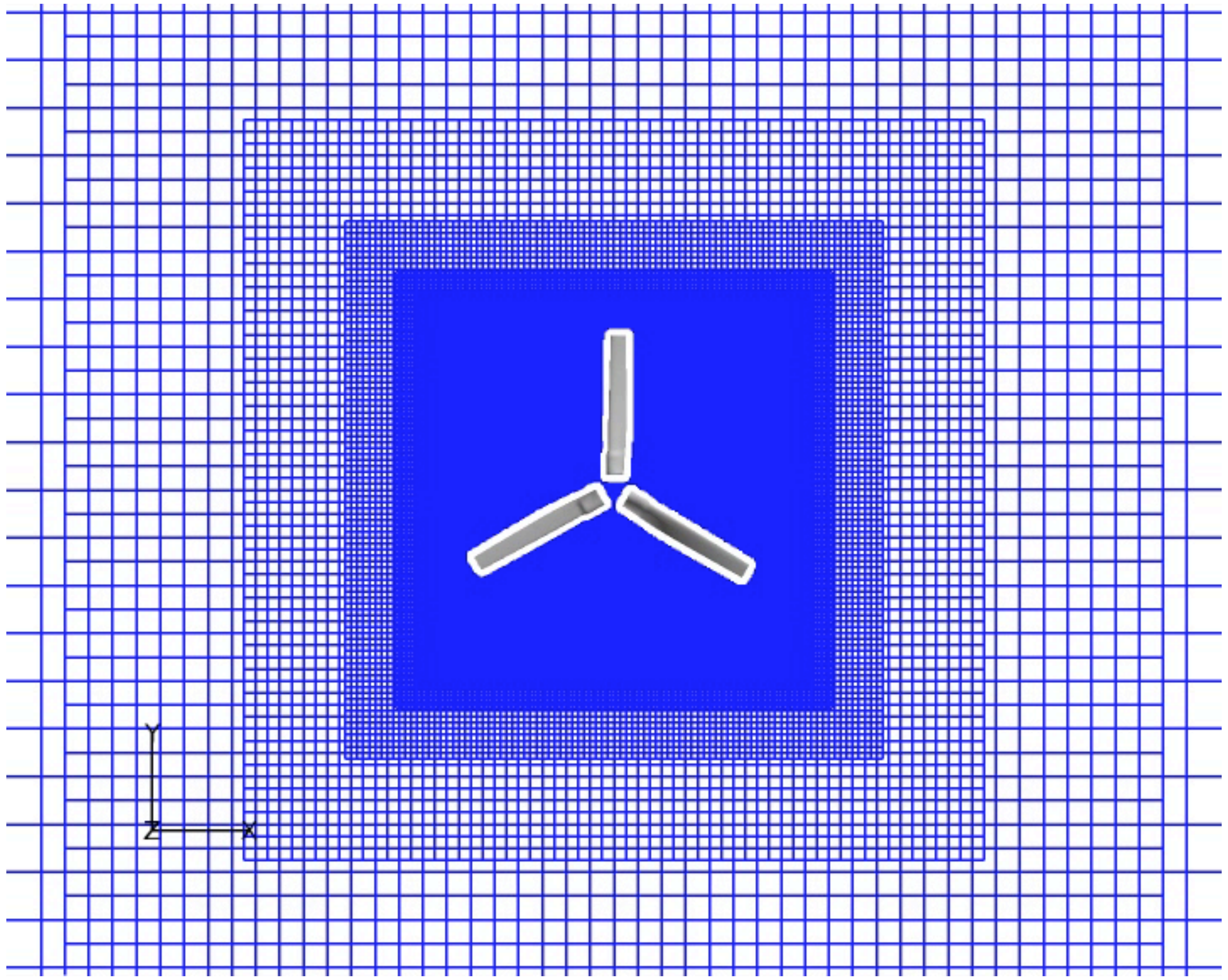
Inviscid Plunging NACA 0012 airfoil at $M_\infty = 0.5$



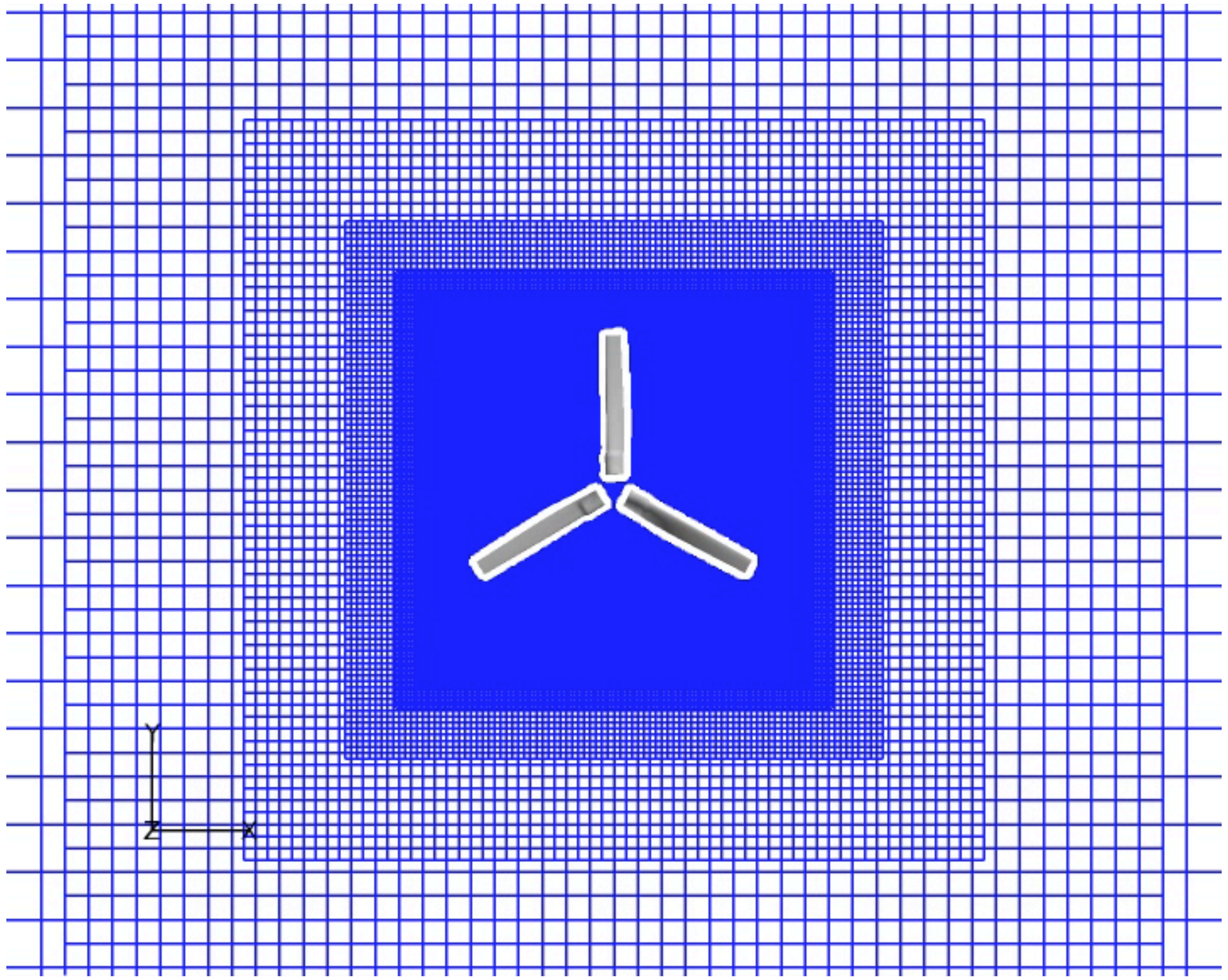
Three-Dimensional V22 TRAM

Hover - $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$, 14 degree collective

Rigid Motion



Relative Motion



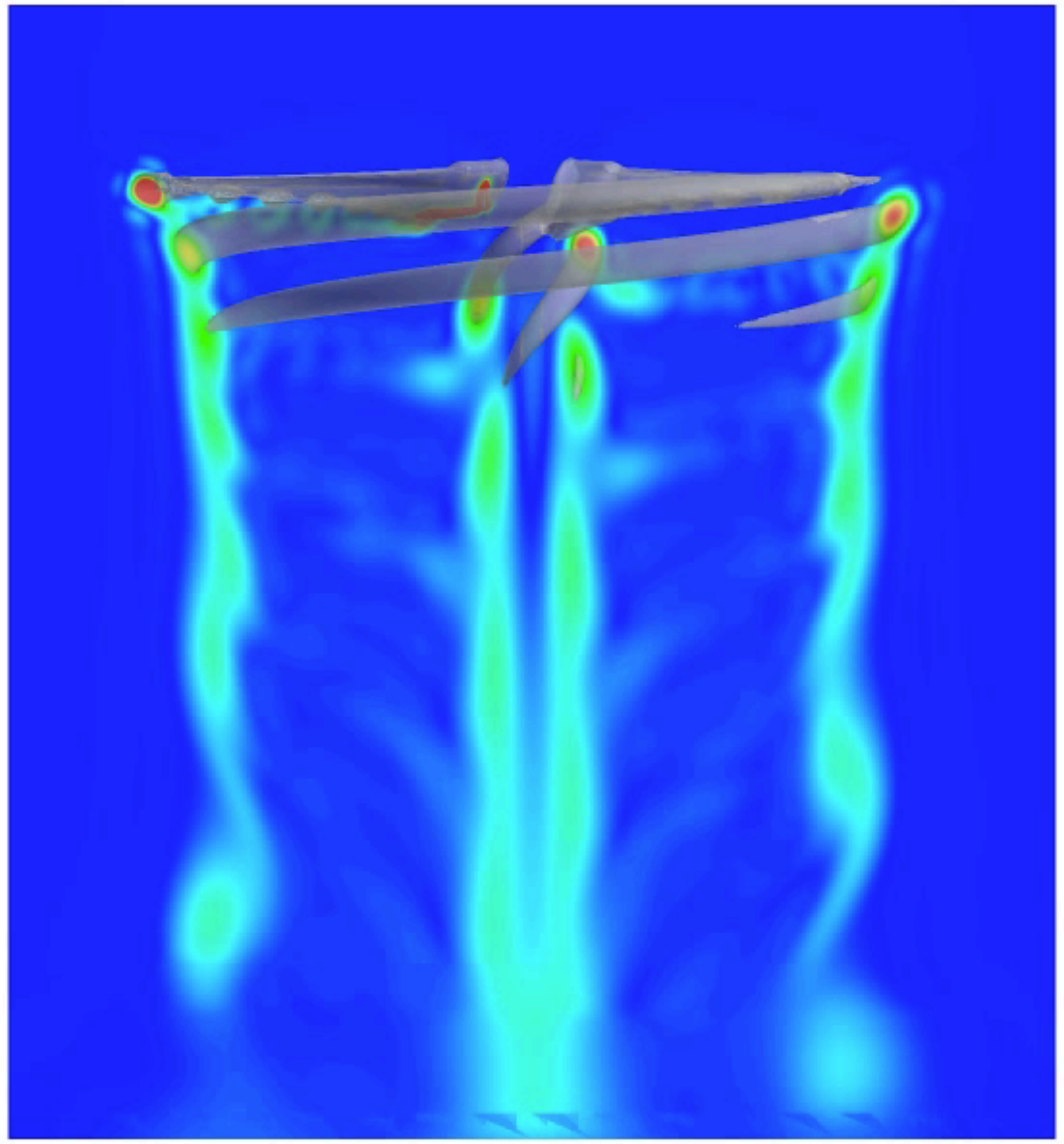
All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Three-Dimensional V22 TRAM

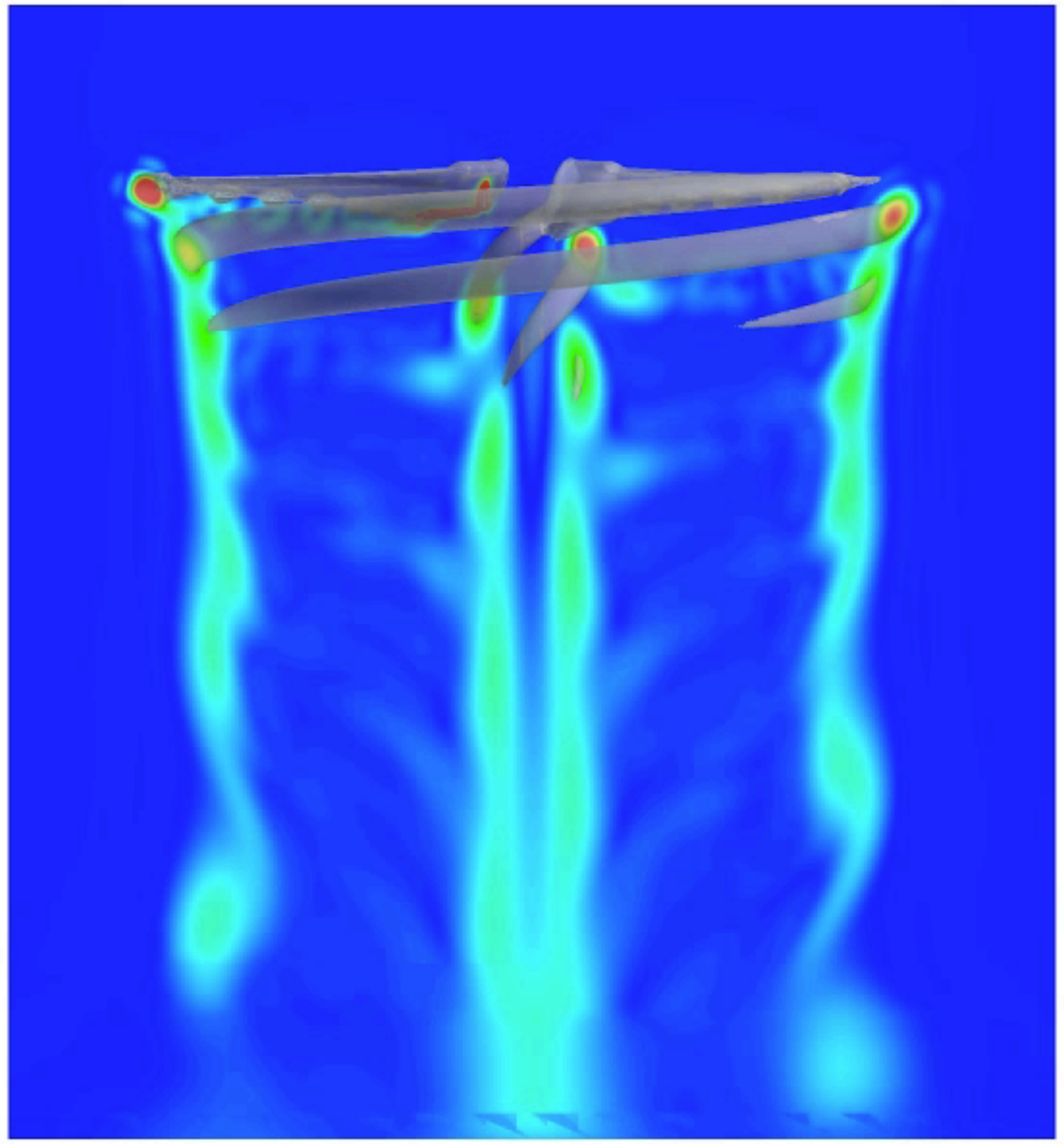
Hover - $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$, 14 degree collective

Vorticity Magnitude

Time Accurate



Time Spectral



Single harmonic, N = 3

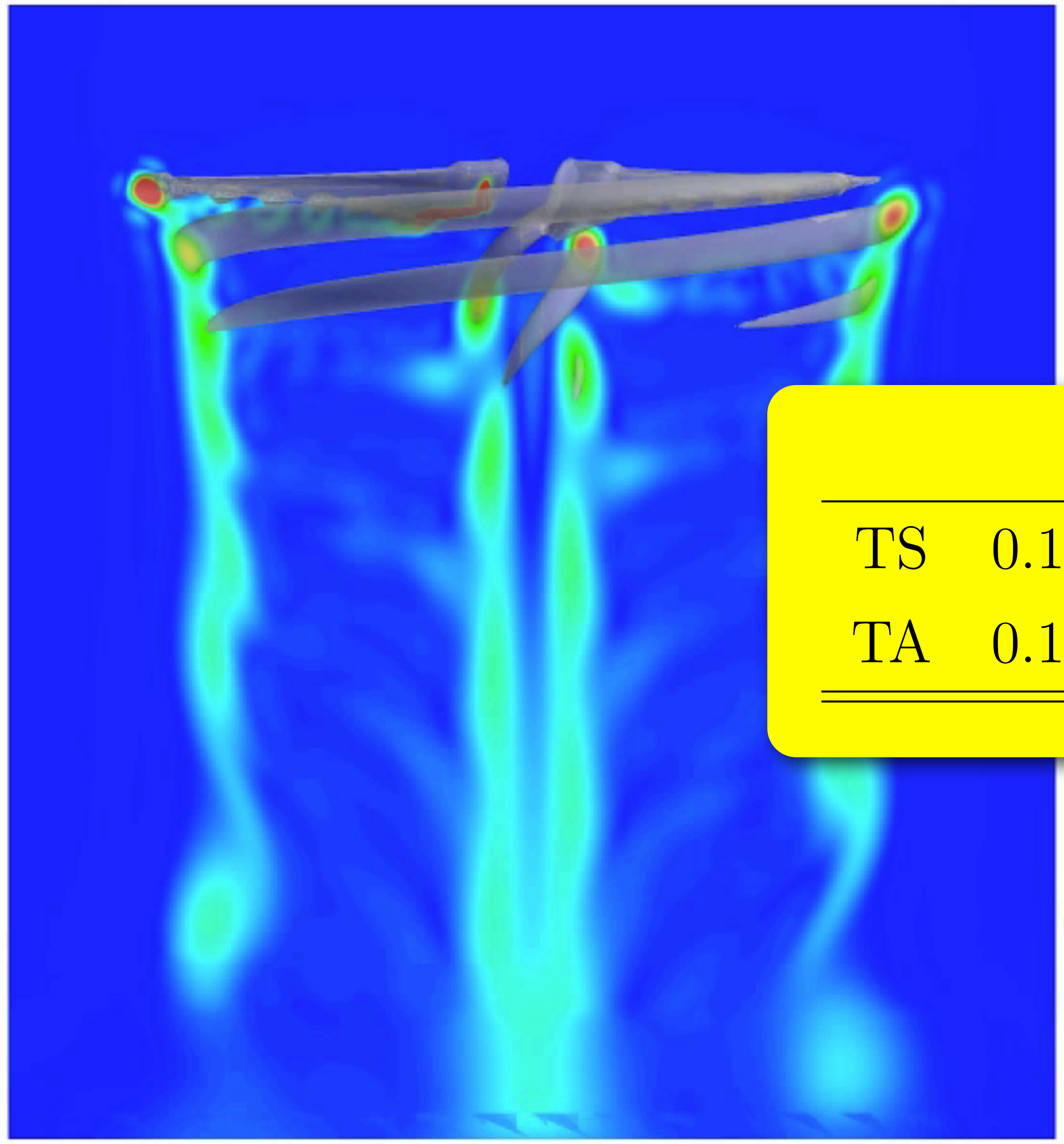
All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Three-Dimensional V22 TRAM

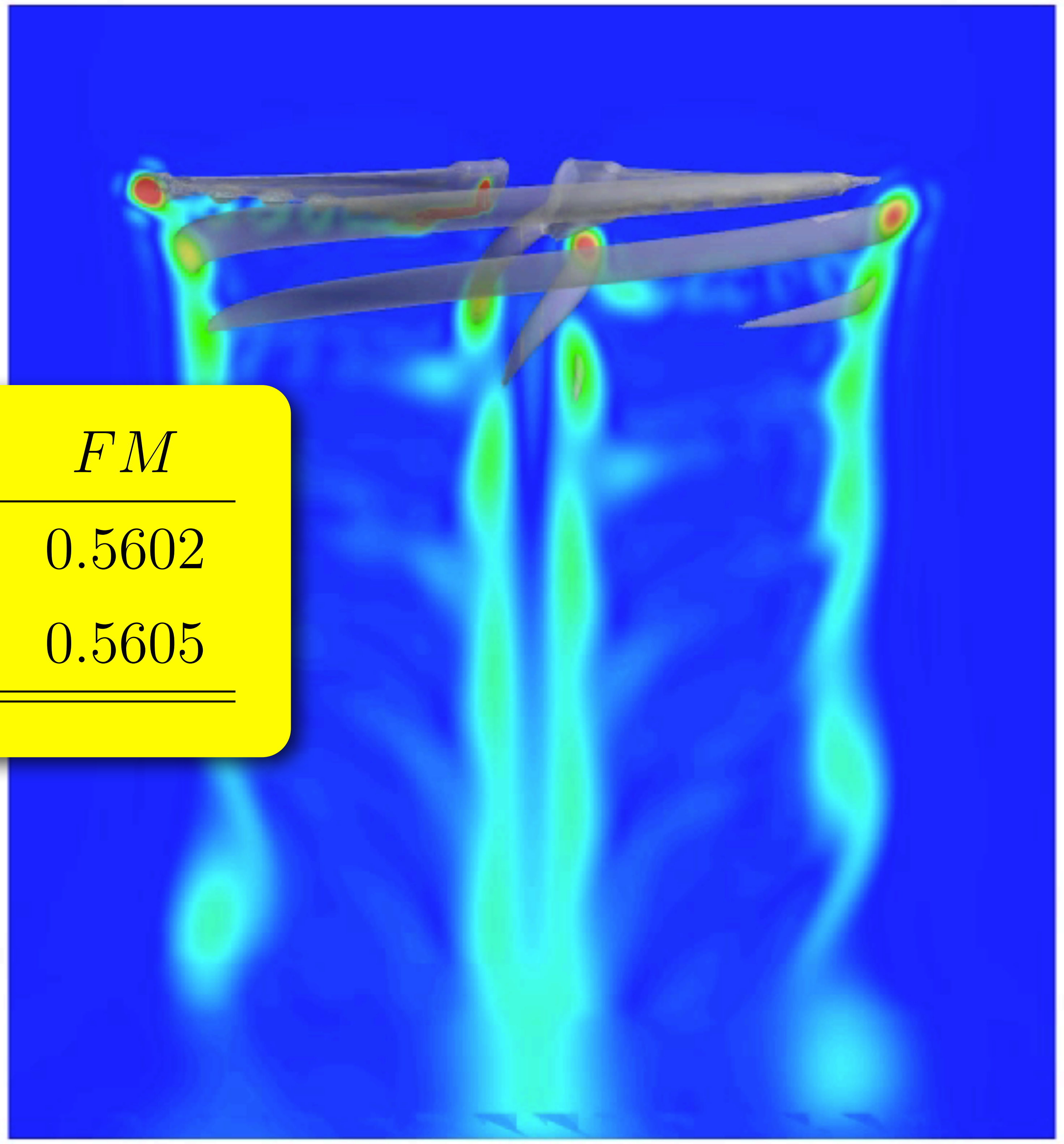
Hover - $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$, 14 degree collective

Vorticity Magnitude

Time Accurate



Time Spectral



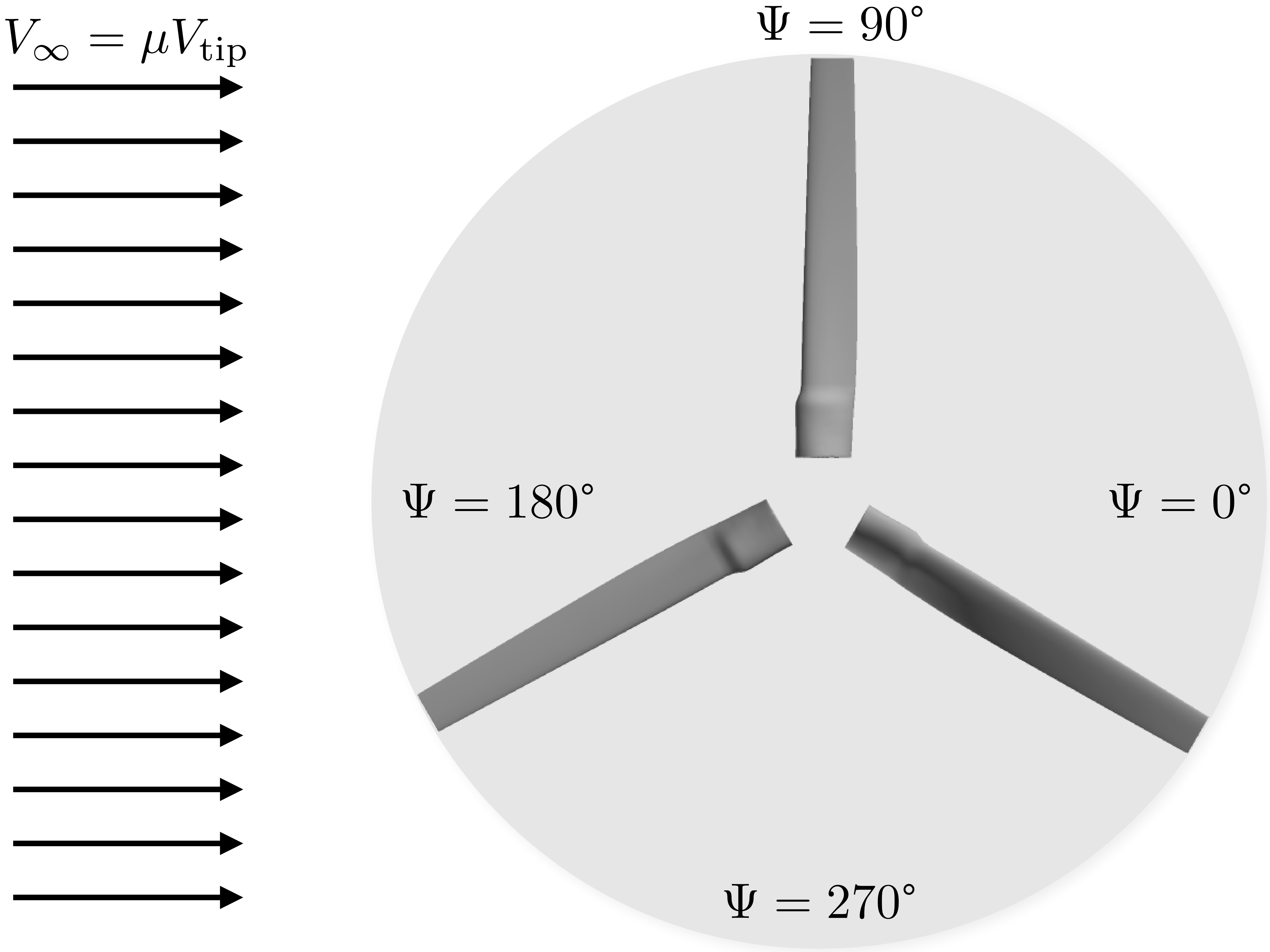
	C_T	C_Q	FM
TS	0.11379×10^{-1}	0.15320×10^{-2}	0.5602
TA	0.11382×10^{-1}	0.15320×10^{-2}	0.5605

Single harmonic, N = 3

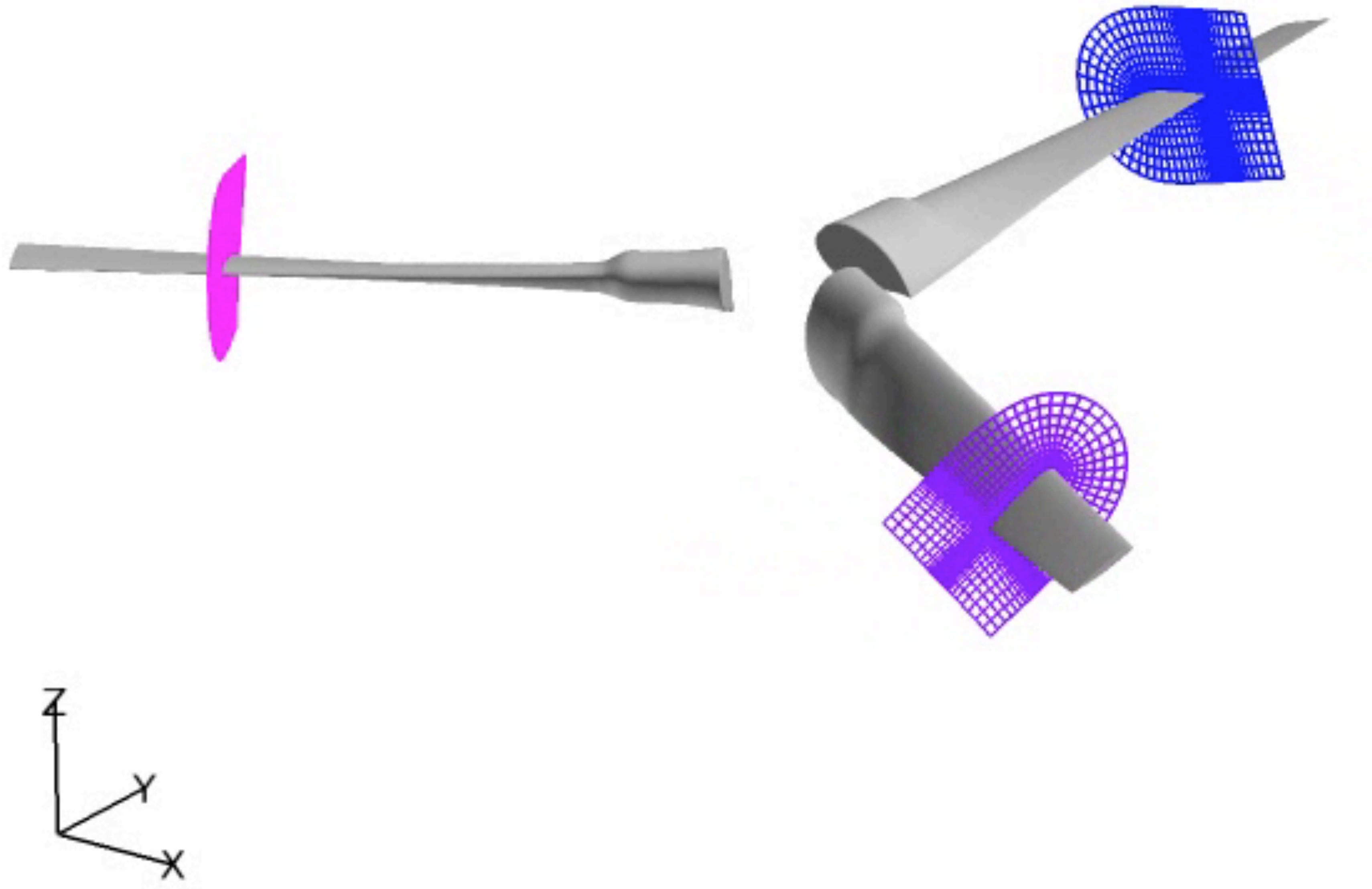
All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Three-Dimensional V22 TRAM

Forward Flight - Advance ratio, μ = 0.2, M_{tip} = 0.625, Re = 2.1 x 10^6



$$\theta\left(\Psi\right) = \theta_0 + \theta_c \cos\left(\Psi\right) + \theta_s \sin\left(\Psi\right)$$



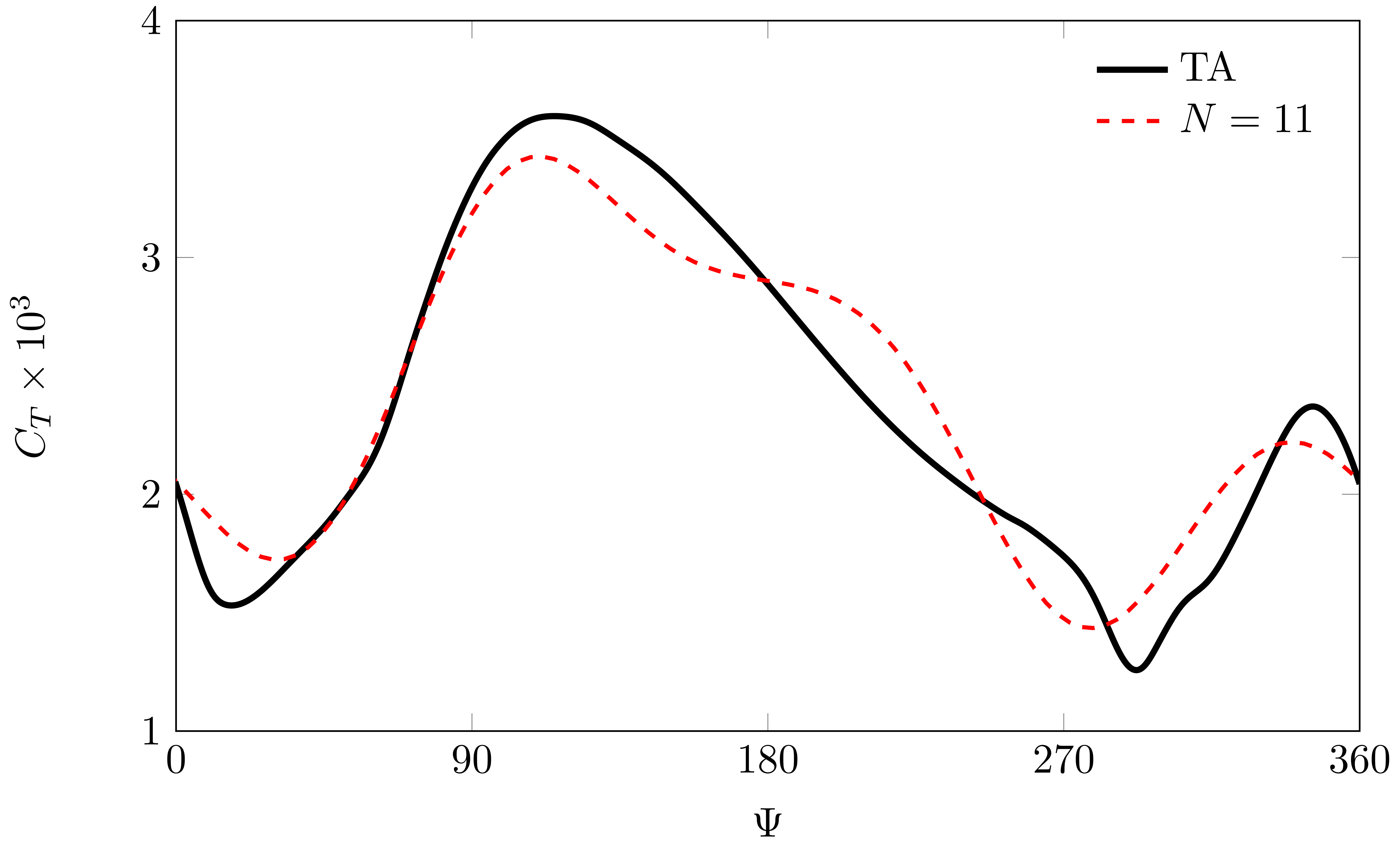
$$\theta_0 = 10.0^\circ, \quad \theta_c = 3.0^\circ, \quad \theta_s = -5.0^\circ$$

All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

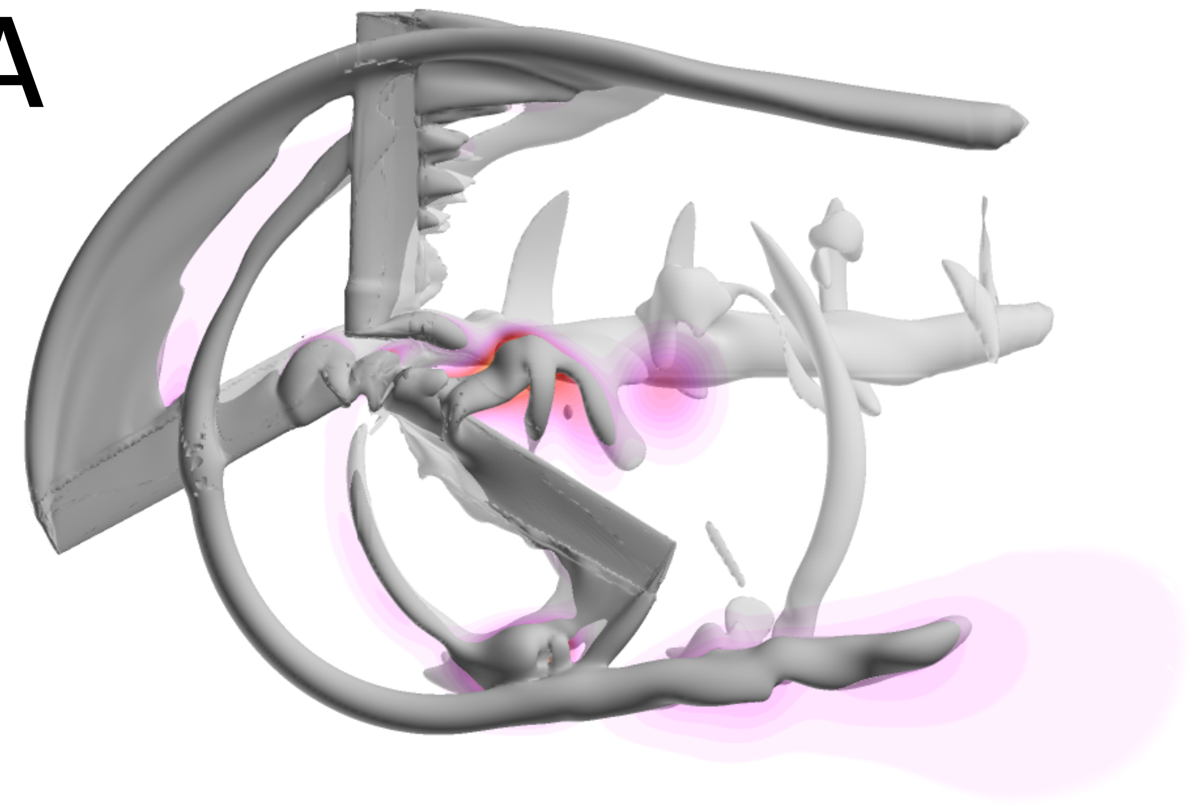
Three-Dimensional V22 TRAM

Forward Flight - Advance ratio, μ = 0.2, M_{tip} = 0.625, Re = 2.1 x 10^6

Convergence of Thrust Coefficient



TA

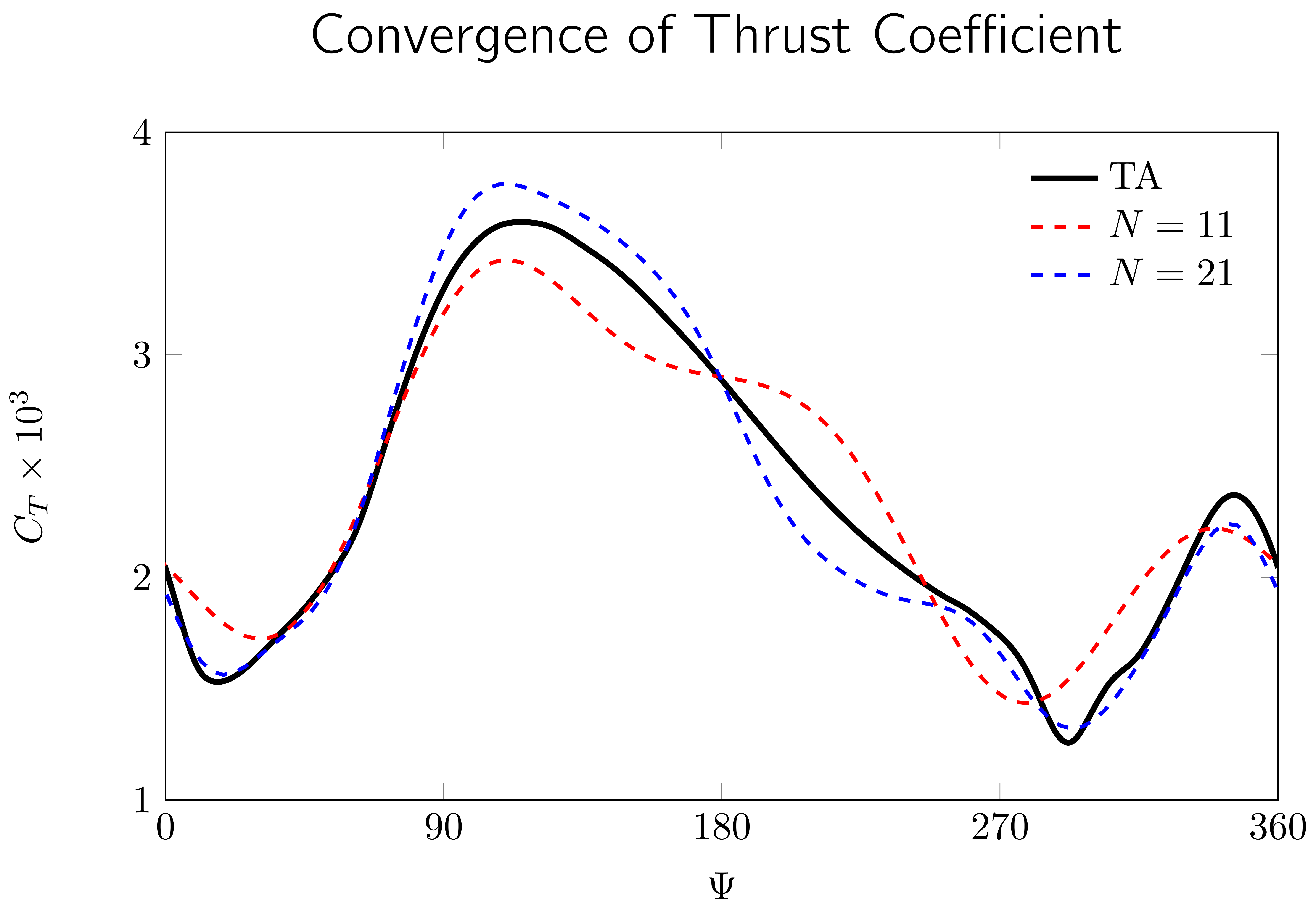


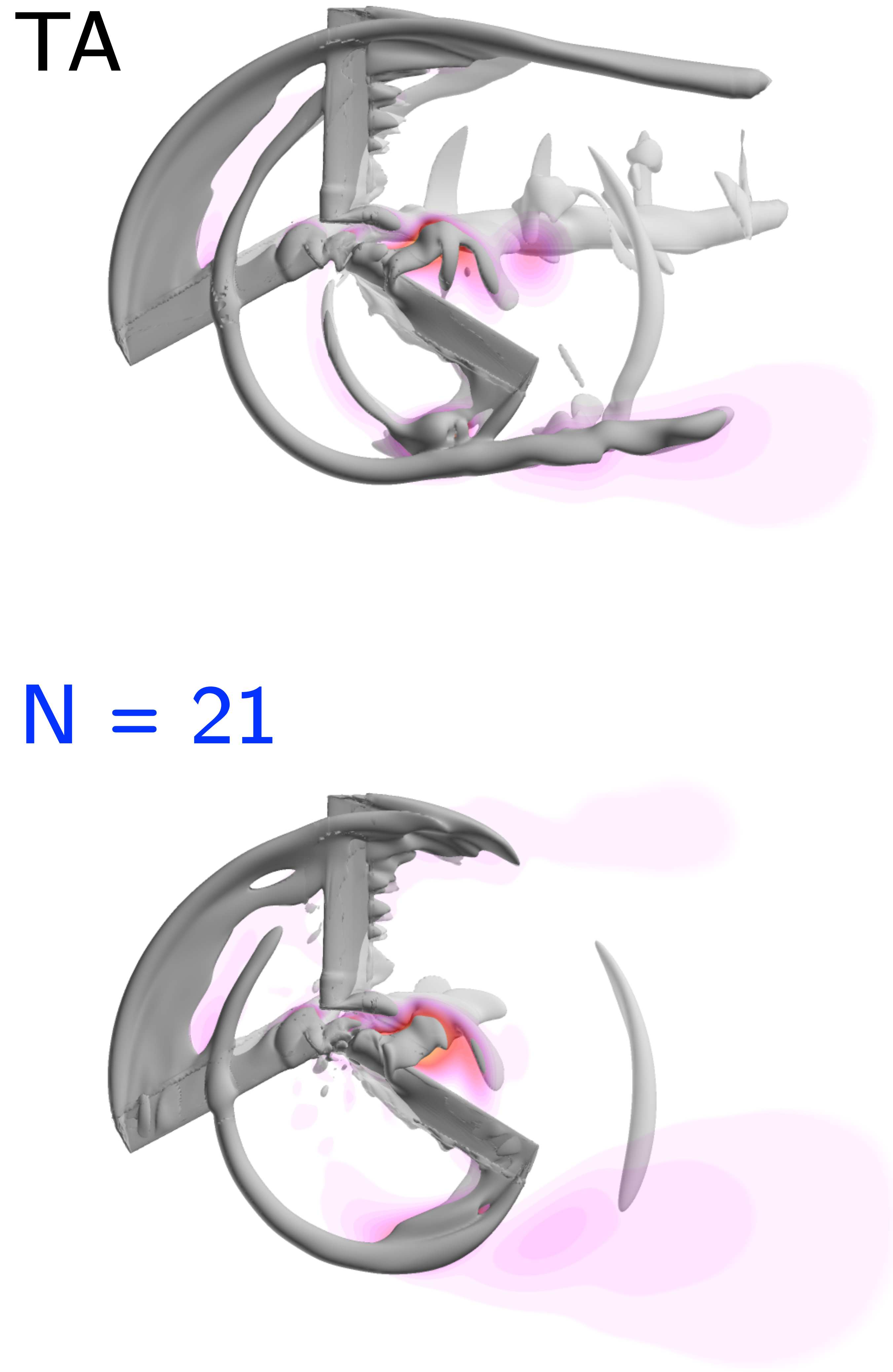
N = 11

All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Three-Dimensional V22 TRAM

Forward Flight - Advance ratio, $\mu = 0.2$, $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$

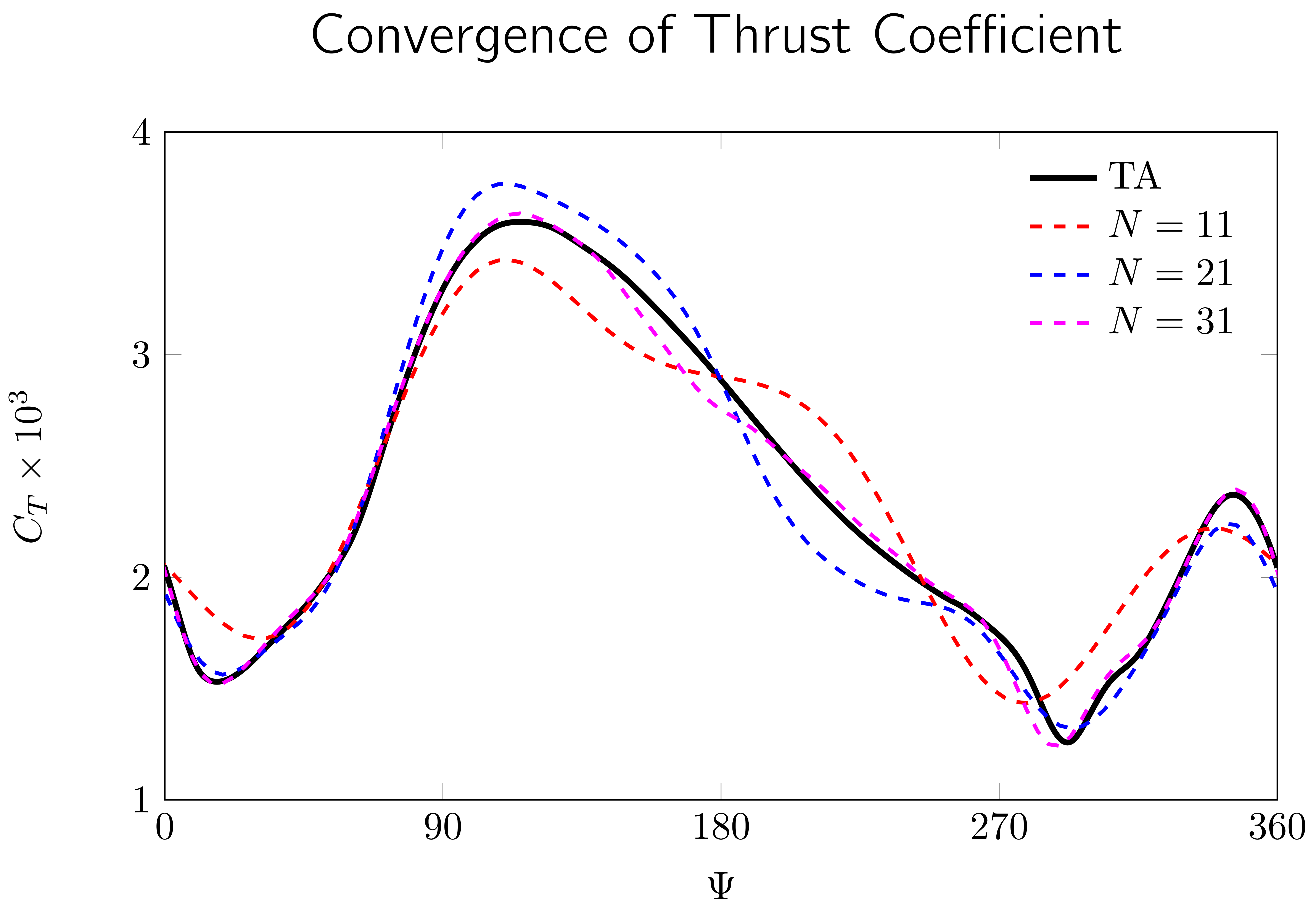


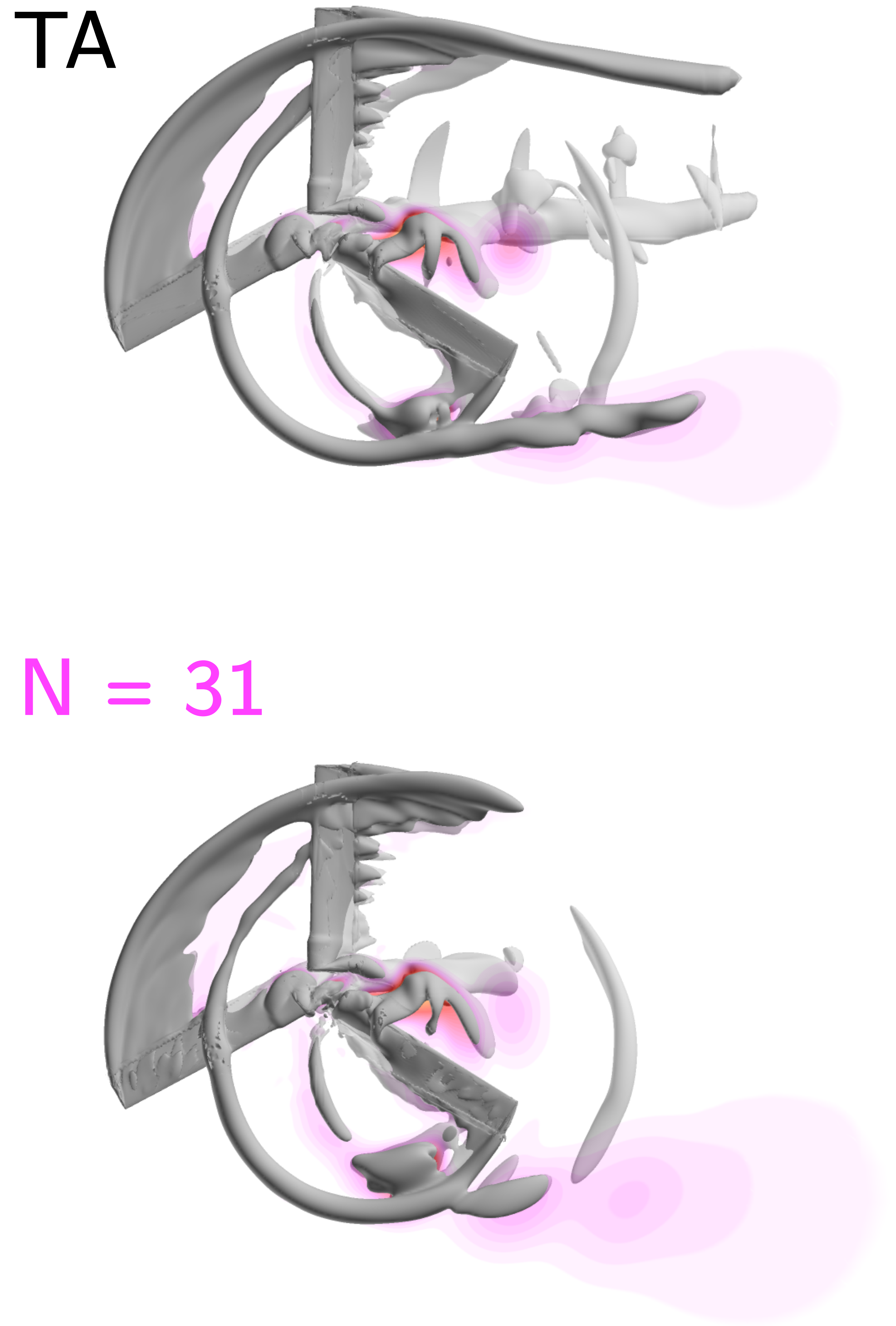


All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Three-Dimensional V22 TRAM

Forward Flight - Advance ratio, $\mu = 0.2$, $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$





All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Three-Dimensional V22 TRAM

Forward Flight - Advance ratio, $\mu = 0.2$, $M_{tip} = 0.625$, $Re = 2.1 \times 10^6$

N = 11 | N = 21 | N = 31 | Time Accurate, N = 1440

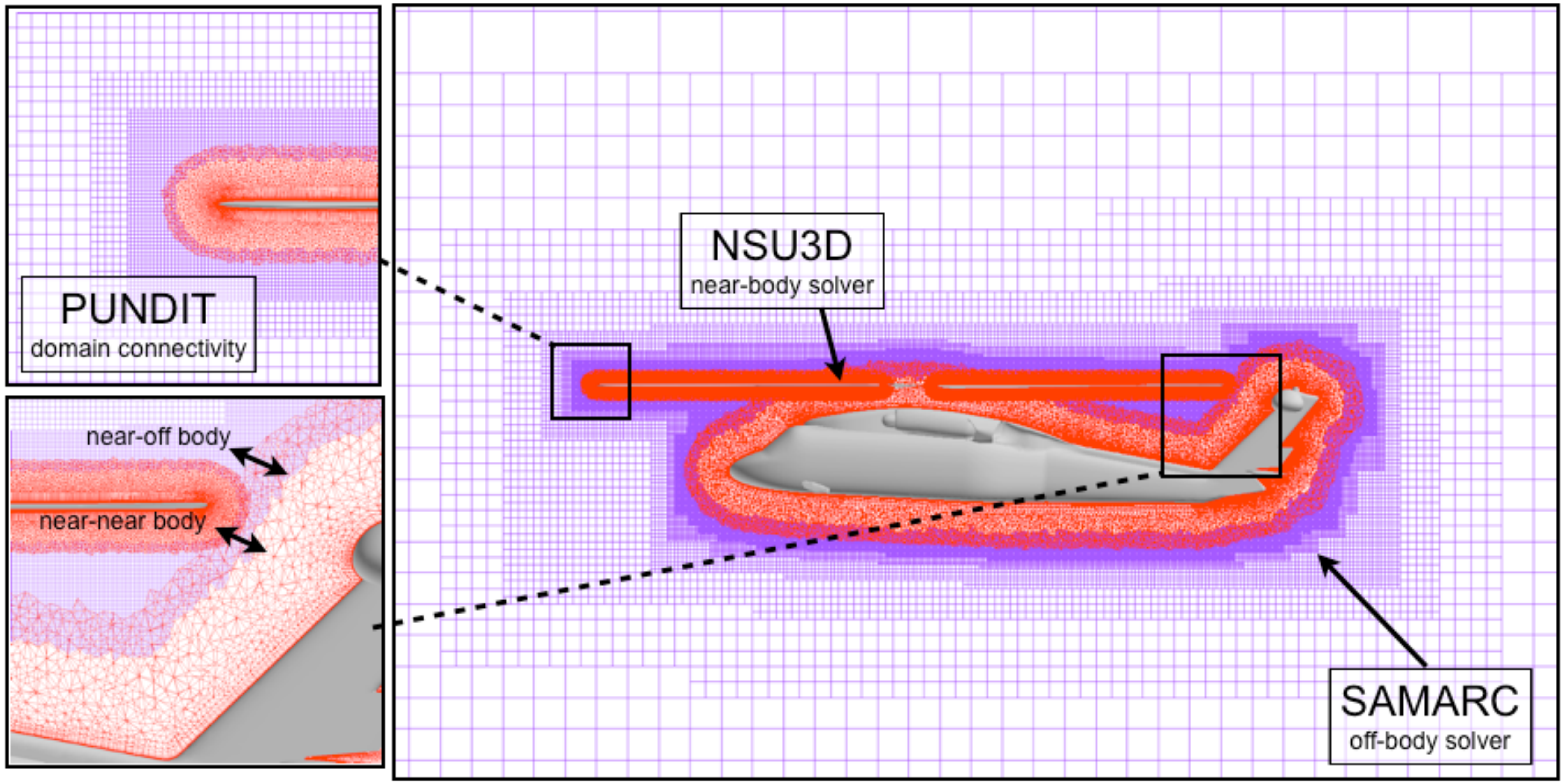
All cases run on 10 20-core Ivy-bridge nodes on Pleiades supercomputer

Future Work

- Space-time multigird
- Parallel in time for periodic problems (e.g. hover or straight & level forward flight)
- Parallel in time for non-periodic problems

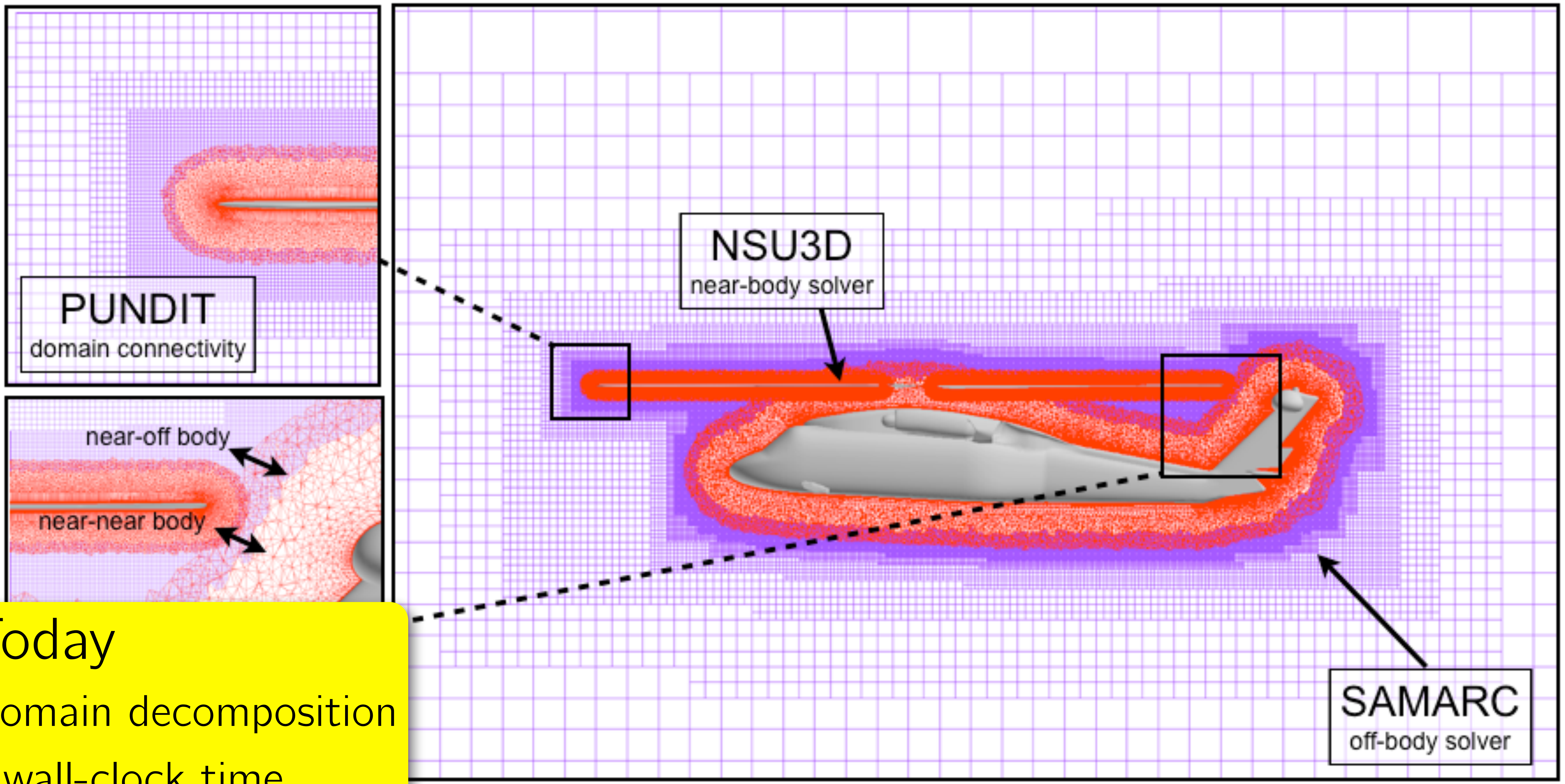
Future Work

- Space-time multigird
- Parallel in time for periodic problems (e.g. hover or straight & level forward flight)
- Parallel in time for non-periodic problems



Future Work

- Space-time multigird
- Parallel in time for periodic problems (e.g. hover or straight & level forward flight)
- Parallel in time for non-periodic problems



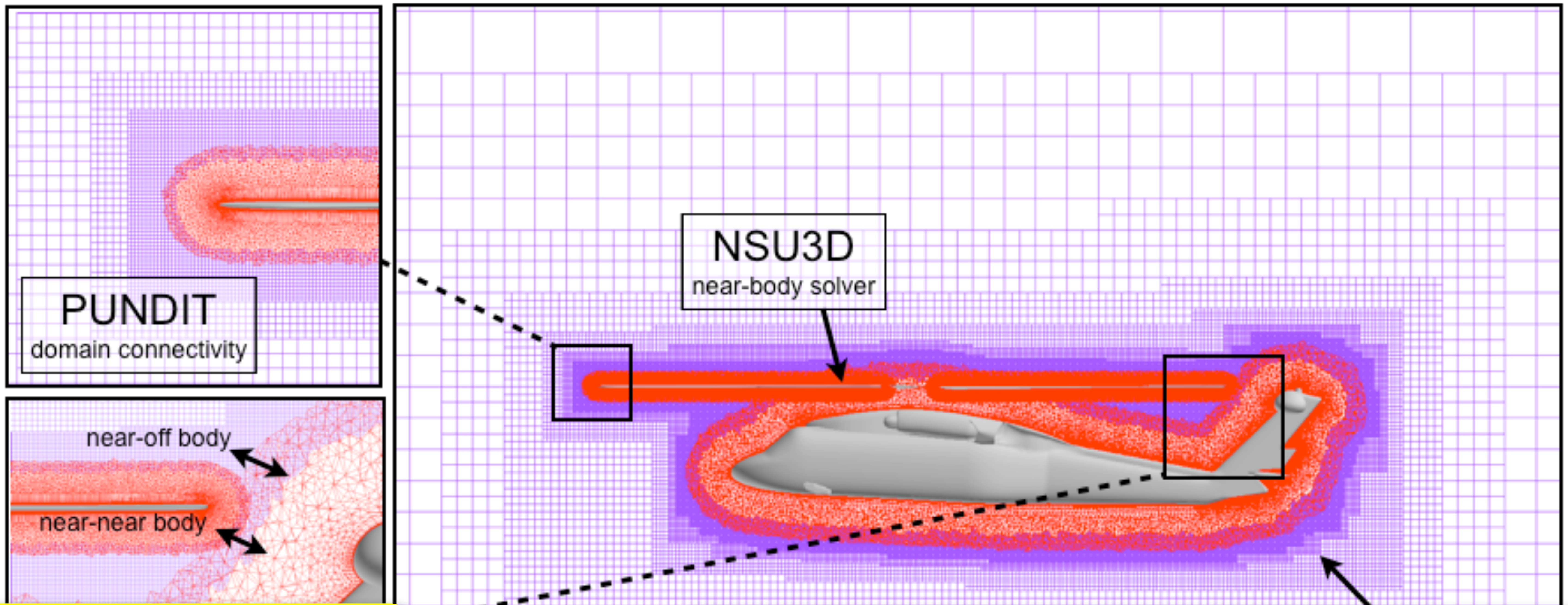
Today

1000 CPUs for domain decomposition

∼30 hours wall-clock time

Future Work

- Space-time multigird
- Parallel in time for periodic problems (e.g. hover or straight & level forward flight)
- Parallel in time for non-periodic problems



Today

1000 CPUs for domain decomposition

∼30 hours wall-clock time

Tomorrow

1,000 CPUs in space x 1,000 CPUs in time = 1M processors

∼<u>5</u> minutes wall-clock time

Acknowledgements

US Army Aeroflightdynamics Directorate for financial & technical support

NASA Advanced Supercomputing (NAS) Division for computational resources & technical support